103. *Open Mappings and Metrization Theorems*

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Let X be a T_1-space and let $\mathfrak{U}=\bigcup_{n=1}^{\infty}\mathfrak{U}_n$ be an open base of X where each $\mathfrak{U}_n$ is a point-finite system of open sets, then $\mathfrak{U}$ is called to be a σ-point-finite open base of X.

In this note, we shall obtain the necessary and sufficient condition that X has a σ-point-finite open base which is a generalization of K. Nagami's theorem [7]. As its application, we shall next obtain some metrization theorems.

1. Open images. K. Nagami [7] has shown the following theorem: *a metric space is always an open compact image*[1] *of a 0-dimensional metric space.* As a generalization of this theorem, we get the following

Theorem 1. *A T_1-space X has a σ-point-finite open base if and only if X is an open compact image of a 0-dimensional metric space.*

Proof. As the "if" part is easily seen from our previous note ([4], Theorem 5), we shall prove the "only if" part.

The following proof is carried out in the similar way as K. Nagami [7]. We may assume that X has a σ-point-finite open base $\mathfrak{U}=\bigcup_{n=1}^{\infty}\mathfrak{U}_n$ such that each $\mathfrak{U}_n=\{U_{\alpha_n}\mid \alpha_n\in A_n\}$ is a point-finite open covering of X and $\mathfrak{U}_{n+1}$ is a refinement of $\mathfrak{U}_n$ for $n=1,2,\cdots$. Let A be the set of points $a=(\alpha_n;\ n=1,2,\cdots)$ of the product space $\prod_{n=1}^{\infty}A_n$, where each A_n is a discrete topological space, such that $\bigcap_{n=1}^{\infty}U_{\alpha_n}=x$ for any point x of X. Then A is a 0-dimensional metric space as the subspace of $\prod_{n=1}^{\infty}A_n$. Let $f(a)=x$, then f is an open continuous mapping of A onto X such that $f^{-1}(x)$ is compact for any point x of X (cf. [7]). This completes the proof.

As an immediate consequence of Theorem 1 and a theorem in our previous note ([4], Theorem 5), we get the following

Theorem 2. *A T_1-space X has a σ-point-finite open base if and only if there exists a countable family $\{\mathfrak{U}_n\}$ of point-finite open coverings of X such that $\{(S(x,\mathfrak{U}_n)\mid n=1,2,\cdots\}$ is a neighborhood basis of x for each point x of X.*

1) Let $f(X)=Y$ be an open continuous mapping. If $f^{-1}(y)$ is compact for each point y of Y, then Y is said to be an open compact image of X.