137. On Algebraic Varieties Uniformizable by Bounded Domains

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Let D be a bounded domain in C^n and Γ a discontinuous group of analytic automorphisms of D. We suppose throughout this paper (unless otherwise mentioned) that Γ satisfies the following two conditions:

(1) No element of Γ except the identity has a fixed point in D.

(2) $V=\Gamma\backslash D$ is compact.

Then V is, as is well-known, analytically equivalent to a non-singular projective algebraic variety. (Cf. [2], [5] Theorem 3 or [6] Theorem 6.) We consider in the following algebraic varieties V, expressible in the form $\Gamma\backslash D$, where D and Γ are as described above.

$\mathfrak{A}(V)$ will denote the group of all analytic automorphisms of V, $K(V)$ the field of all meromorphic functions on V, and $\mathfrak{A}(K(V))$ the group of all automorphisms of $K(V)$ over C.

Igusa [5] proved, in case D is a hypersphere in C^n:

$$|z_1|^2+\cdots+|z_n|^2<1,$$

that (a) V is a minimal model, and (b) $\mathfrak{A}(K(V))$ is a finite group.

We shall show that (i) (a) holds in general (for any bounded domain D), (ii) $\mathfrak{A}(V)$ is finite, if D is simply-connected, and (iii) $\mathfrak{A}(V) \cong\mathfrak{A}(K(V))$. (ii), (iii) imply of course (b). To prove (i) and (iii), we can utilize the idea of [5] (Theorems 6 and 7, p. 675), and the result (ii) can be found in [3], [7]. However we shall also give a proof of (ii) for completeness' sake.

It is well-known that any compact Riemann surface V of genus ≥ 2 can be uniformized by $D=$ the unit disc of the complex plane. In this case, the fact that $\mathfrak{A}(V)$ is finite is a classical Schwarz-Klein theorem, and it is also known that $\mathfrak{A}(V)\cong\mathfrak{A}(K(V))$. Thus our results contain generalizations of these classical facts.

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§1. A complex manifold M imbedded in a projective space will be called "minimal" if any meromorphic mapping from any complex manifold U into M is necessarily holomorphic. Here a meromorphic mapping is defined invariantly using the inhomogeneous coordinates of M and any local coordinates of U. (Cf. [5], p. 674.)

Theorem 1. *Notations being as above, the variety* $V=\Gamma\backslash D$ *is*