156. On Bochner Transforms. II

A Generalization Attached to M(n, R) and "an" n-Dimensional Bessel Function

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1. Introduction. The concept of Bochner transforms is a generalization of Fourier transforms of radial functions. (Bochner [1] and Iwasaki [2].) In this paper we shall define Bochner transforms attached to the space of matrices $M(n, \boldsymbol{R})$ and investigate some of its properties. As an analogy to the case of the one-dimensional Euclidean space we get "n-dimensional Bessel functions". We shall give Bessel differential equations for these functions.

Probably the Bochner transforms have a close relation to Siegel modular functions. We shall discuss in this direction elsewhere.

2. Definitions and notations. We denote by $\boldsymbol{P}_0=\boldsymbol{P}_0(n, \boldsymbol{R})$ the space of non-negative symmetric matrices of degree n, by $\boldsymbol{P}$ the set of strictly positive elements in $\boldsymbol{P}_0$ and M_k the space of continuous functions on $\boldsymbol{P}_0$ which is C^∞ on $\boldsymbol{P}$, invariant by the automorphism of $\boldsymbol{P}_0$, $x\to{}^tuxu$ where $u\in U=\boldsymbol{O}(n, \boldsymbol{R})$, and $\int_{\boldsymbol{P}}(\det x)^{\frac{k}{2}}|\varphi(x)|^2dx$ is convergent. Now

Definition. *The Bochner transform $T=T^n_{\lambda,k}$ is a linear operator on M_k which satisfies the following conditions* (B):

(B$_1$) *the function* $\varepsilon(x)=\exp\left(-\frac{2\pi}{\lambda}\operatorname{tr} x\right)$ *is mapped to itself by T,*

(B$_2$) $\int_U\varphi({}^tw{}^tuxuw)du$ *with $\varphi\in M_k$ and $w\in GL(n, \boldsymbol{R})$ is mapped by T, as a function of x, to*

$$\int_U T\varphi(w^{-1}\,{}^tuxu{}^tw^{-1})du\cdot|\det w|^{-k}\quad\begin{pmatrix}du \text{ is the Haar measure on } U\\ \text{normalized by } \int_U du=1\end{pmatrix},$$

(B$_3$) $$\int_{\boldsymbol{P}}(\det x)^{\frac{k}{2}}\varphi(x)\psi(x)dx=\int_{\boldsymbol{P}}(\det x)^{\frac{k}{2}}T\varphi(x)T\psi(x)dx,$$

where $\varphi, \psi\in M_k$ and dx is a measure on $\boldsymbol{P}$ invariant by $x\to{}^twxw$ (see [2]).

Any element φ of M_k is a spherical function on $\boldsymbol{P}$, therefore it has the Fourier transform in Gelfand-Selberg sense. On our stand point it may be called the *Mellin transform* of φ and it is defined as follows (Selberg [3] pp. 56–59):