1. On Bochner Transforms. III

Case of $\mathfrak{p}$-adic Number Fields

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1. In the following we shall consider *Bochner transforms* attached to matrices algebras over $\mathfrak{p}$-adic number fields.

Let k be a completion of a finite algebraic number field with respect to a finite prime ideal $\mathfrak{p}$, $\mathfrak{o}$ the ring of integers in k, π a prime element of k and $\mathfrak{u}$ the unit group. We denote by A, O, G, and U the matrices algebra $M(n, k)$, the order $M(n, \mathfrak{o})$, the group $GL(n, k)$ and the unit group of O respectively. Let $\mathcal{F}$ mean the space of the all U-biinvariant continuous functions integrable on A.

Definition. *The Bochner transform $T=T_k^n$ is a linear operator on $\mathcal{F}$ which satisfies the following conditions* (B):

(B_1') *the characteristic function $\varepsilon(x)$ of O is mapped to itself by T,*

(B_2) *as a function of x, $\int_U \varphi(xuw)du$ with $\varphi\in\mathcal{F}$ and $w\in G$ is mapped to $\int_U T\varphi(xu^tw^{-1})du\,|\det w|_{\mathfrak{p}}^{-k}$ by T* (du is the Haar measure of U normalized by $\int_U du=1$),

(B_4) *there is a U-biinvariant continuous function $\alpha(x)$ on O such that*

$$\int_O \alpha(x)\varphi(x)\,dx=\int_O \alpha(x)\,T\varphi(x)\,dx$$

for any function $\varphi\in\mathcal{F}$ (see [3]).

Remarks. (i) The function $\varepsilon(x)$ in (B_1') corresponds to the function $e^{-\pi x^2}$ in (B_1) of [3] as $\mathfrak{p}$-component of the function defined on the adele ring appeared in the proof of the functional equation of Riemann zeta-function in the thesis of Tate [4].

(ii) Condition (B_4) is an analogy of the *modular relation* (Bochner [2]). On the stand point of Bochner-Chandrasekharan it may be better to consider integrals on an arbitrary compact set. But we treat only the analogy of ordinary modular forms.

(iii) Using the zonal spherical function

$$\omega(w;\, s)=\omega(w;\, s_1, s_2,\cdots, s_n)=\int_U \Big|\prod_{i=1}^n t_i(wu)\Big|_{\mathfrak{p}}^{-s_i+(i-1)}du,$$

where $t_i(x)$ is the i-th diagonal element of the upper trigonal part