45. The Role of Mollifiers in Wightman Functions

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§ 1. Introduction. The normal representation of Canonical commutation relations can be treated at least in three different ways [1]. Weyl's formulation and Segal's formulation are one of them.

The unitary operators which is constructed by Weyl's formulation is used for the construction of Wightman functions. These functions are one example of Wightman functions related to the axiomatic relativistic quantum field theory [2].

For the reinvestigation of the meaning of the unitary operator constructed by Weyl's formulation the exact definition of multiplication of the operator valued distributions is needed. But its concept is not clear. Here we define the three kinds of multiplication and show the method to find the most natural one.

The meaning of multiplication of field functions has not been definite. In the present paper, the meaning of non local field functions which appeared in Wightman functions above will be made clear by using our definitions. Furthermore, by using our definition, the meaning of commutation relations becomes clear.

On the other hand, the orthogonality between the domain of the free Hamiltonian and that of the total Hamiltonian is already proved in the case of the neutral scalor field with fixed point source [3-5]. Nobody yet has constructed a linear compatible topology satisfying the Hausdorff's axioms (A-D) in which the statevectors in interaction field can be approximated by the sequence of the state vectors in free field [6-7].

In Wightman's method the domain of the free Hamiltonian and that of the total Hamiltonian can be constructed separately in the same way. This situation is not sufficient in clarifying the relation between these two domains.

If we try to clarify this point forcibly, then it can be seen that the lack of the sufficiently strong linear compatible topology is an essential obstacle [9]. Hence, only free fields have been taken up so far. The case of interacting field will be referred to briefly, in this paper.

Here we shall show the limitation for the usual Gelfand's construction by using Weyl's formulation and show the effects of mollifiers upon this construction.

 \S 2. The multiplication of operator valued distributions. We