108. Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. XIII

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Preliminaries. Let $\{\lambda_\nu\}_{\nu=1,2,3,\dots}$ be an arbitrarily prescribed bounded infinite sequence of complex numbers (counted according to the respective multiplicities), and let $D_1, D_2, \cdots, D_n$ be arbitrarily prescribed, bounded, connected, and closed sets with (linear or planar) positive measures in the complex plane such that they are mutually disjoint and each of them does not contain any point belonging to the closure of $\{\lambda_\nu\}$. Then there are infinitely many bounded normal operators N such that the point spectrum and the continuous spectrum of each N of them are given respectively by $\{\lambda_\nu\}$ and the union of one, D_j, of $D_1, D_2, \cdots, D_n$ and the set of all those accumulation points of $\{\lambda_\nu\}$ which do not belong to $\{\lambda_\nu\}$ itself, as can be found from Theorem 29. If we suppose that $\{\varphi_\nu^{(j)}\}_{\nu=1,2,3,\dots}$ and $\{\psi_\mu^{(j)}\}_{\mu=1,2,3,\dots}$ are arbitrarily given incomplete orthonormal sets orthogonal to each other such that the complex abstract Hilbert space $\mathfrak{H}$ under consideration is determined by themselves, then one of those bounded normal operators in $\mathfrak{H}$, which will be denoted by N_j, is expressible in the form of

$$N_j=\sum_{\nu=1}^{\infty}\lambda_\nu\varphi_\nu^{(j)}\otimes L_{\varphi_\nu^{(j)}}+\sum_{\mu=1}^{\infty}\Psi_\mu^{(j)}\otimes L_{\psi_\mu^{(j)}},$$

where $\Psi_\mu^{(j)}=\sum_{\kappa=1}^{\infty}(N_j\psi_\mu^{(j)}, \psi_\kappa^{(j)})\psi_\kappa^{(j)}$; and moreover, if we denote $(N_j\psi_\mu^{(j)}, \psi_\kappa^{(j)})$ by $\beta_{\mu\kappa}$ for brevity of expression, the matrix-operator $(\beta_{\mu\kappa})$ associated with the infinite matrix where $\beta_{\mu\kappa}$ is the element appearing in row μ column κ is a bounded normal operator with $\sum_{\kappa=1}^{\infty}|\beta_{\mu\kappa}|^2>|\beta_{\mu\mu}|^2>0$ $(\mu=1, 2, 3, \cdots)$, as we have already demonstrated in the preceding paper. Let now $\{K^{(j)}(\lambda)\}$ be the complex spectral family of N_j; let $\{\psi_{\mu_p}^{(j)}\}_{p=1,2,3,\dots}(\subset\{\psi_\mu^{(j)}\}_{\mu=1,2,3,\dots})$ be the incomplete orthonormal set determining the subspace $K^{(j)}(D_j)\mathfrak{H}$; let $\mathfrak{M}_1$ be the subspace determined by $\{\varphi_\nu^{(1)}\}_{\nu=1,2,3,\dots}$; let $\mathfrak{N}_j$ and $\mathfrak{N}_j'$ be the subspaces determined by $\{\psi_\mu^{(j)}\}_{\mu=1,2,3,\dots}$ and $\{\psi_{\mu_p}^{(j)}\}_{p=1,2,3,\dots}$ respectively; let $f_{1\alpha}$ and $f_{1\alpha}'$ $(\alpha=1, 2, 3, \cdots, m)$ be arbitrary elements consisting of all $\varphi_\nu^{(1)}$ in $\mathfrak{M}_1$; let $f_{2\alpha}$ and $f'_{2\alpha}$ be arbitrary elements consisting of all $\psi_\mu^{(1)}$ in $\mathfrak{N}_1$; let $g_{j\beta}$ and $g_{j\beta}'$ $(j=1, 2, 3, \cdots, n;$ $\beta=1, 2, 3, \cdots, k_j)$ be arbitrary elements consisting of all $\psi_{\mu_p}^{(j)}$ in $\mathfrak{N}_j'$; and let

$$\chi(\lambda)=\sum_{\alpha=1}^{m}((\lambda I-N_1)^{-\alpha}(f_{1\alpha}+f_{2\alpha}),\ (f_{1\alpha}'+f_{2\alpha}'))+\sum_{j=2}^{n}\sum_{\beta=1}^{k_j}((\lambda I-N_j)^{-\beta}g_{j\beta}, g_{j\beta}'),$$