101. On Boundary Value Problem for Parabolic Equations

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(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 12, 1964)

1. Introduction. Let us consider the parabolic equation

$$(1) \qquad \frac{\partial}{\partial t}u=Au \quad \text{in} \quad (0,T)\times\Omega$$

$$\left(A=\sum_{|\nu|\leqq 2b}a_\nu(t,x)\left(\frac{\partial}{\partial x}\right)^\nu,\ L=\frac{\partial}{\partial t}-A\right)$$

with the zero initial data and the general boundary data

$$(2) \qquad B_j u=f_j\ (j=1,\cdots,b) \quad \text{on} \quad (0,T)\times S$$

$$\left(\beta_j=\sum_{|\nu|\leqq r_j}b_{j\nu}(t,x)\left(\frac{\partial}{\partial x}\right)^\nu,\ 0\leqq r_j\leqq 2b-1\right),$$

where Ω is a domain in R^n surrounded by a hypersurface S.

Recently, this problem was treated by Eidelman for systems ([1]). Here we use his construction and estimates of kernels in the case of constant coefficients and Ω is a half space. We shall introduce an operator defined on the boundary which plays an analogous role to the Riemann-Liouville-operator which was used by Mihailov in one dimensional case ([2]), therefore we need not assume that all r_j coincide, which was assumed by Eidelman in case of non-convex region. Finally we have the estimates for the Green function.*)

I thank Prof. Mizohata very much for his kind advices and encouragements throughout this subject.

Now, let $\{\overline{V}\}_I$ be a finite covering of S and a point $x=(x_1,\cdots,x_n)$ of $\overline{V}$ be represented by a local coordinate $\bar{x}'=(\bar{x}_1,\cdots,\bar{x}_{n-1})$, such that $x_j=F_j(\bar{x}')$ $(j=1,\cdots,n)$, where $F_j(\bar{x}')$ is of class-C^s $(s=2b+1+\gamma,\gamma>0)$, and $\bar{\bar{x}}'=\bar{\bar{x}}'(\bar{x}')$ is class-C^s where $x\in\overline{V}\cap\overline{\overline{V}}$. Then we have a n-dimensional neighbourhood $\overline{U}\supset\overline{V}$, such that the transformation defined by $x_j=F_j(\bar{x}')+N_j(\bar{x}')\dot{x}$ $(j=1,2,\cdots,n)$ is one-to-one and of class-c^{s-1} between $x\in\overline{U}$ and $\bar{x}$, where $N_x=(N_1,\cdots,N_n)$ is the unit inner normal vecter at $x\in S$. Here we put $\widetilde{S}=\bigcup_I\overline{U}$.

Put $A_0(\eta+zN_x;t,x)=(-1)^b\sum_{|\nu|=2b}a_\nu(t,x)(\eta+zN_x)^\nu$ and $B_{0j}(\eta+zN_x;t,x)=(i)^{r_j}\sum_{|\nu|=r_j}b_{j\nu}(t,x)(\eta+zN_x)^\nu$, where $\eta\in T_x=R^n/\{zN_x\}$, $z\in R^1$, $t\in(0,T)$, $x\in S$. Let $A_{0+}(p,\eta,z;t,x)$ be the polynomial of z of degree b (the coefficient of z^b is 1), where the roots are composed of all the roots z of $p-A_0(\eta+zN_x;t,x)=0$, having the positive imaginary part. Then let us denote

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*) Detailed proof will be published in a forthcoming paper.