100. Markovian Systems of Measures on Function Spaces

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A Markovian process defined on a path space is a system of non-negative probability measures on a function space. In this note we construct systems of signed measures corresponding to contraction semigroups (Theorem 1). These systems can be regarded as a generalization of Markovian processes. It is well known that the generator of a continuous Markovian process on a Euclid space is a generalized elliptic differential operator of second order. An analogous result holds also in our cases (Theorem 2).

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1. Let (E, ρ) be a σ-compact metric space, and $\boldsymbol{C}$ be the Banach space consisting of all real-valued, continuous, and bounded functions on E with the uniform norm $\|\cdot\|$. Let T_t be a strongly continuous contraction semigroup on $\boldsymbol{C}$. We assume that the operators T_t are expressed in the integral form:

$$T_t f(x)=\int_E f(y)P(t,x,dy) \quad (f\in \boldsymbol{C}),$$

where $P(t,x,\cdot)$ are signed measures[1] which satisfy the Kolmogoroff-Chapman equation

$$P(t+s,x,\cdot)=\int_E P(t,x,dy)P(s,y,\cdot).$$

Let ∂ be an extra point added to E and put

$$\widetilde{P}(t,x,\cdot)=\begin{cases} P(t,x,E\frown\cdot)+\delta_\partial(\cdot)\{1-P(t,x,E)\}, & \text{if } x\in E,\\ \delta_\partial(\cdot) & \text{, if } x=\partial,\end{cases}$$

where δ_∂ is the Dirac measure. Then $\widetilde{P}(t,x,\cdot)$ are measures on $E\smile\partial$, which satisfy the Kolmogoroff-Chapman equation and also the equality

$$(1)\qquad \widetilde{P}(t,x,E\smile\partial)=1.$$

We assume in the following that the function 1 belongs to the domain $\mathfrak{D}(\mathcal{G})$ of the generator $\mathcal{G}$ of the semigroup T_t. We have

$$(2)\qquad |\widetilde{P}|(t,x,E\smile\partial)\leqq e^{\gamma t},\text{[2]}$$

where $\gamma=\|\mathcal{G}1\|$.

Let Ω be the set of all functions that are defined on $[0,\infty)$ and take values from $E\smile\partial$. We write

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1) Hereafter we omit the adjective "signed".

2) By $|\widetilde{P}|$ we denote the total variation of $\widetilde{P}$.