139. *On Differentiability in Time of Solutions of Some Type of Boundary Value Problems*

By Hiroki TANABE

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1. **Introduction.** The differentiability problem of the solutions of the abstract differential equation

$$du(t)/dt+A(t)u(t)=f(t)$$

in a Banach space was treated by S. Agmon and L. Nirenberg ([2]) quite generally when $A(t)$ does not depend on t. A. Friedman [4] generalized some of their results to the equations in a Hilbert space in which $A(t)$ may depend on t. However he assumes that the domain of $A(t)$ does not depend on t, therefore his theorem cannot be applied directly to the boundary value problem

$$\partial u(t, x)/\partial t+A(t, x, D_x)u(t, x)=f(t, x), \quad x\in\Omega, \tag{0.1}$$

$$B_j(t, x, D_x)u(t, x)=0, \quad x\in\partial\Omega, \quad j=1, \cdots, m, \tag{0.2}$$

where $D_x=(\partial/\partial x_1, \cdots, \partial/\partial x_n)$ and $A(t, x, D_x)$ is an elliptic operator of order $2m$ in a bounded domain $\Omega\subset R^n$ for each t, unless the coefficients of $B_j(t, x, D_x)$, $j=1, \cdots, m$, are independent of t. The object of the present note is to show that A. Friedman's method can be applied to the problem (0.1)–(0.2) when the positive and negative imaginary axes are of minimal growth with respect to the system $A(t, x, D_x)$, $\{B_j(t, x, D_x)\}_{j=1}^m$ in the sense of S. Agmon [1], and hence that the solution of (0.1)–(0.2) is smooth in t as a function with values in $L^2(\Omega)$ or $H_{2m}(\Omega)$ if $f(t, x)$ and the coefficients of $A(t, x, D_x)$, $B_j(t, x, D_x)$, $j=1, \cdots, m$, are sufficiently smooth.

2. **Preliminary lemmas.** Let Ω be a bounded domain with a smooth boundary in R^n. By $H_k(\Omega)$ we denote the set of all measurable functions in Ω whose distribution derivatives of order up to k are square integrable, the norm of $H_k(\Omega)$ being denoted by $\|\ \|_k$.

Assumptions. (I) For each $t\in(-\infty, \infty)$ $A(t, x, D_x)=\sum_{|\alpha|\leq 2m} a_\alpha(t, x)D_x^\alpha$ is an elliptic operator of order $2m$ in $\bar{\Omega}$.

(II) $\{B_j(t, x, D_x)\}_{j=1}^m=\{\sum_{|\beta|\leq m_j} b_{j\beta}(t, x)D_x^\beta\}_{j=1}^m$ is a normal system of boundary operators for each t. The order m_j of $B_j(t, x, D_x)$ is smaller than $2m$ and does not depend on t.

(III) $$\pm(-1)^m iD_y^{2m}-A(t, x, D_x) \tag{1.1}$$

is elliptic with respect to (x, y) in the cylindrical domain $\Omega\times\{y; -\infty<y<\infty\}$ for each fixed t. The Complementing Condition is satisfied by (1.1) and $\{B_j(t, x, D_x)\}_{j=1}^m$ in $\Omega\times\{y; -\infty<y<\infty\}$ for each t.

(IV) The coefficients of $A(t, x, D_x)$ as well as those of $\{B_j(t, x,$