# 159. *Remarks on Ninomiya's Domination Principle*

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1. **Introduction.** In the $n$-dimensional Euclidean space $R^n$ $(n\geq 1)$, the potential of a given order $\alpha$, $0<\alpha<n$, of a measure $\mu$ in $R^n$ is defined by

$$U_\alpha^\mu(x)=\int |x-y|^{\alpha-n}d\mu(y),$$

provided the integral on the right exists. The kernel $|x-y|^{\alpha-n}$ will be called the kernel of order $\alpha$. Let $\mu$ be a measure in $R^n$. When the integral

$$\iint |x-y|^{\alpha-n}d\mu(y)d\mu(x)$$

exists, we shall call it the *$\alpha$-energy* of $\mu$. We shall denote the inner capacity of a set $A$ with respect to the kernel of order $\alpha$ by $C_\alpha(A)$. A property is said to hold *$\alpha$-p.p.p.* on a subset $X$ in $R^n$, when the property holds on $X$ except a set $E$ with $C_\alpha(E)=0$. The measure $\mu$ in $R^n$ will be said to be *$\alpha$-finite*, when the potential $U_\alpha^\mu(x)$ is defined and finite *$\alpha$-p.p.p.* in $R^n$. We shall denote the support of a measure $\mu$ in $R^n$ by $S_\mu$.

Ninomiya [3] proved the following domination principle.

In $R^n$ $(n\geq 3)$, let $\alpha$ be a positive number such that $0<\alpha\leq 2$, let $\mu$ be a positive measure with compact support such that the $\alpha$-energy is finite, and let $\nu$ be a positive measure in $R^n$. If

$$U_\alpha^\mu(x)\leq U_\alpha^\nu(x) \qquad \text{on } S_\mu,$$

then

$$U_\beta^\mu(x)\leq U_\beta^\nu(x) \qquad \text{in } R^n$$

for any $\beta$ such that $\alpha\leq\beta<n$.

He proved that the same domination principle is valid in $R^2$ if $0<\alpha\leq 1$. In this paper, we shall prove Ninomiya's domination principle in a possibly general form.

2. **Ninomiya's domination principle. Lemma.**[1] *In $R^n$ $(n\geq 1)$, let $\alpha$ be a positive number such that $0<\alpha\leq 2$, $0<\alpha<2$ or $0<\alpha<1$ according to $n\geq 3$, $n=2$ or $n=1$. Then the kernel of order $\alpha$ satisfies the balayage principle with respect to the kernel of order $\beta$ for any $\beta$ such that $\alpha\leq\beta<n$, namely, for any $p$ in $R^n$ and any closed set $F$, there exists a positive measure $\lambda$, supported by $F$, such that*

$$U_\alpha^\lambda(x)=|x-p|^{\beta-n} \qquad \alpha\text{-p.p.p. on } F,$$

1) Ninomiya (Theorem 2 in [3]) showed this when $n\geq 3$ and $F$ is compact.