155. On Closures of Vector Subspaces. I

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1. Let E be a vector space, and let M be an infinite dimensional vector subspace of E. In a previous paper,[1] we stated a condition which ensures the existence of a locally convex metrizable topology on E possessing the following properties:

(1) M is dense in E;

(2) The induced topology on M is finer than a given locally convex metrizable topology on M.

Moreover, as a consequence of it, we obtained a condition which ensures the existence of a locally convex metrizable topology on E satisfying merely the requirement (1). The main interest in the present paper is on the requirement (1), and we shall concern, in what follows, with the problem of existence of a locally convex Hausdorff topology on E possessing the property (1) without the restriction that the topology is metrizable.

The terminology and notations used in the previous paper will be continued in this paper.

2. Throughout this section the operation of polar will be taken in the dual system (E^{**}, E^*).

We have immediately the following lemmas.

LEMMA 1. *Let E be a vector space, and let E' be a vector subspace of E^*. If the dual system (E, E') is separated, then E'° is a $\sigma(E^{**}, E^*)$-closed vector subspace of E^{**} contained in an algebraic supplement of E in E^{**}. Conversely, if F is a $\sigma(E^{**}, E^*)$-closed vector subspace of E^{**} such that $E\frown F=\{0\}$, then the dual system (E, F°) is separated.*

LEMMA 2. *Let E be a locally convex vector space, and let E' be its dual. For every vector subspace M of E, the following conditions are equivalent:*

(1) M is dense in E.

(2) $M^{\circ}\frown E'=\{0\}$.

(3) The vector subspace $M^{\circ\circ}+E'^{\circ}$ of E^{**} is dense in E^{**} for the weak topology $\sigma(E^{**}, E^*)$.

Thus we have

THEOREM 1. *Let M be a vector subspace of a vector space E.*

1) S. Kasahara: Locally convex metrizable topologies which make a given vector subspace dense. Proc. Japan Acad., **40**, 718-722 (1964).