176. An Approach to Locally Convex Topological Linear Spaces*)

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In this note, I shall explain some ideas to study locally convex topological linear spaces and locally m-convex topological algebras.[1] The detailed results will be published in future.

An inconvenient point to discuss a locally convex topological linear space E is that, in general, there exists a fundamental family of infinitely many semi-norms $\{p_\alpha\}$ on E to define the topology. To eliminate such an inconvenient point and to develop a new theory, consider the mapping $x \to ||x|| = \{\cdots, p_\alpha(x), \cdots\}$ with the weak topology, then we have a continuous mapping $x \to ||x|| \in \prod_\alpha R^1$, where R^1 is the set of real numbers. The order in $\prod_\alpha R^1$ is defined by the coordinate-wise order.

Therefore, we have a kind of norm $||x||$ which takes on values in $\prod_\alpha R^1$, and $||x||=0$ implies $x=0$, since $\{p_\alpha\}$ is fundamental. Hence we have the following proposition.

For any locally convex topological linear space E, there is a norm $||x||$ which takes on values in a product space of the real line. The norm satisfies the following conditions:

1) $||x|| \geq 0$, $||x||=0$ if and only if $x=0$,
2) $||x+y|| \leq ||x|| + ||y||$,
3) $||\lambda x|| = |\lambda| \, ||x||$, where λ is a scalar.

Further, any locally m-convex topological algebra (in the sense of E. A. Michael [2]) admits a norm satisfying the conditions 1), 2), 3) and $||xy|| \leq ||x|| \, ||y||$. For a non-Archimedean topological linear spaces by A. F. Monna [4], we also have a similar result: Let E be a non-Archimedean topological linear space (in the sense of A. F. Monna), then there is a norm $||x||$ taking on values in a product space of the real line satisfying

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1) I introduced these considerations, when I gave my lecture course (the 2nd semester of 1963) on topological linear spaces at the Universidad del Sur, Bahia Blanca, Argentina. Further, I spoke of these topics and some results in my lectures at Montevideo, Sao Paul, and Refice Universities, and the IMPA at Rio de Janeiro. On the other hand, by the kind suggestion of Professor Yu. Smirnov, I knew a similar discussion by Tashkent group (see [6]), though their articles are not accessible to me. For a brief summary, see Yu. Smirnov [5].