# 168. *Special Type of Separable Algebra over a Commutative Ring*

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In the previous paper [4], we considered a type of separable algebra over a field which has the simple ideal components whose degrees are all prime to the characteristic of the field. In this paper we consider the case of algebra over a commutative ring.

Let $\Lambda$ be an algebra over a commutative ring $R$. In the enveloping algebra $\Lambda^e=\Lambda\otimes_R\Lambda^0$ we consider the involution $*$ defined by $(x\otimes y^0)^*=y\otimes x^0$ for $x\otimes y^0\in\Lambda^e$. We set $J=\{x\otimes 1^0-1\otimes x^0 \mid x\in\Lambda\}$, then $J^*=J$. Let $A$ be the right annihilator of $J$ in $\Lambda^e$, then $A^*$ is the left annihilator of $J$ and a left ideal in $\Lambda^e$. Let $\varphi:\Lambda^e\rightarrow\Lambda$ be the $\Lambda^e$-homomorphism defined by $\varphi(x\otimes y^0)=xy$, then $\varphi(A^*)$ is a two sided ideal of $\Lambda$. In this paper we shall call $\Lambda$ a strongly separable algebra over $R$ when $\varphi(A^*)=\Lambda$.

In § 1, we shall show that $\Lambda$ is a strongly separable algebra over $R$ if and only if $\Lambda$ is a separable algebra over $R$ and $\Lambda=C\oplus[\Lambda,\Lambda]$ where $C$ is the center of $\Lambda$ and $[\Lambda,\Lambda]$ is the $C$-submodule of $\Lambda$ generated by $xy-yx$ for all $x,y\in\Lambda$. In § 2, we consider an $R$-algebra $\Lambda$ such that $\Lambda$ is an $R$-projective module, and we shall show that if $A\neq 0$ then there exists a non zero left ideal in $\Lambda$ which is generated by a finite number of elements as $R$-module. Finally, we have that for a central separable $R$-algebra $\Lambda$, $\Lambda$ is hereditary if and only if $R$ is hereditary. In this paper we assume that every rings and algebras have identity elements.

## 1. Strongly separable algebra.

PROPOSITION 1. *Let $\Lambda$ be an algebra over $R$. Then $\varphi(A^*)=\Lambda$ if and only if $\Lambda^e=\Lambda^eJ\oplus A^*$. If $\varphi(A^*)=\Lambda$ then $\Lambda$ is a separable algebra over $R$ and $\Lambda=C\oplus[\Lambda,\Lambda]$, where $C$ is the center of $\Lambda$ and $[\Lambda,\Lambda]$ is the $C$-submodule of $\Lambda$ generated by $xy-yx$ for all $x,y\in\Lambda$.*

*Proof*. If $\Lambda^e=\Lambda^eJ\oplus A^*$ then we have $\varphi(A^*)=\Lambda$. Now we assume $\varphi(A^*)=\Lambda$. Since $\operatorname{Ker}\varphi=\Lambda^eJ$, we have $\Lambda^e=A^*+\Lambda^eJ$. Therefore we have $\Lambda^{e*}=A^{**}+J^*\cdot\Lambda^{e*}$ and $\Lambda^e=A+J\Lambda^e$. Let $1\otimes 1^0=z_1+z_2$ with $z_1\in A$, $z_2\in J\Lambda^e$. If $x\in A^*\cap\Lambda^eJ$ then $x=x\cdot 1\otimes 1^0=xz_1+xz_2=0$. It follows that $A^*\cap\Lambda^eJ=0$ and $\Lambda^e=A^*\oplus\Lambda^eJ$. Thus the first half of the proposition is proved. If $\varphi(A^*)=\Lambda$, then $\varphi$ induces an isomorphism of $A^*$ onto $\Lambda$ therefore $\Lambda$ is a separable algebra over $R$. Since $\Lambda^e=\Lambda^eJ\oplus A^*$, there are orthogonal idempotents $e_1\in\Lambda^eJ$ and