85. *Singular Cut-off Process and Lorentz Covariance*

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§ 1. **Introduction.** Singular cut-off process means to construct a sort of convolution (by A-integral) with A-integrable function or limit of them, namely it is to construct the infinite sum (of usual field functions) which play the same role as the field function suffering the usual cut-off [4] p 822. At the first step let's give the exact definition of it. Let $\psi(x)$ denote the quantized field function with the form

$$\psi(x)=(1/(2\pi)^{3/2})\left\{\int (a^{+}(\vec{k})/\sqrt{2k_0})\cdot \exp i(\vec{k}\cdot\vec{x}-k_0 t)d\vec{k} + \int (a(\vec{k})/\sqrt{2k_0})\cdot \exp(-i)(\vec{k}\cdot\vec{x}-k_0 t)d\vec{k}\right\} \cdots (1),$$

$(k_0=\sqrt{k_1^2+k_2^2+k_3^2+m^2})$, $\rho(\vec{x})$ denote the A-integrable function defined on the nowhere dense perfect set which is equivalent to a smooth function (by the meaning of distribution), and $\{\rho_n(\vec{x})\}$ denote the some fixed sequence of the above A-integrable functions with the limit $\bar{\rho}(\vec{x})$ [3] p 137.

Suppose that $\psi_\rho(x)$ is given by the form (means the infinite sum)

$$\psi_\rho(x)=(1/(2\pi)^{3/2})\left\{\int (a^{+}(\vec{k})/\sqrt{2k_0})\cdot(A)\int \rho(\vec{x}')\exp i\{\vec{k}(\vec{x}-\vec{x}')-k_0 t\}d\vec{x}'d\vec{k} + \int (a(\vec{k})/\sqrt{2k_0})\cdot(A)\int \rho(\vec{x}')\exp(-i)\{\vec{k}(\vec{x}-\vec{x}')-k_0 t\}d\vec{x}'d\vec{k}\right\} \cdots (2).$$

Definition 1. The operation constructing $\psi_\rho(x)$, (or $\lim_{n\to\infty}\psi_{\rho_n}(x)$) from $\psi(x)$ is called singular cut-off process.

The representation of cut-off (by using mollifier) and the above representations (1), (2) have a sort of ambiguities deduced from the lack of the exact definition of integral. Here, we use the Λ inhomogeneous Lorentz covariance defined in § 4 Def. 3 as Lorentz covariance. The judgement whether this Lorentz covariance is satisfied or not for three dimensional cut-off also depends on the interpretation of the problem related to the exact definition of δ function and being alike to "arrow's" paradox by Zenon. This judgement is related to the contradiction of the interpretation of rigid body in relativity theory [9] p 176. Though Λ inhomogeneous Lorentz covariance is the more weak condition than the usual Lorentz covariance, the cut-off (three dimensional case) by using smooth function's mollifier exerts the negative influence even on this. This negative influence is based on the lack of Haar measure in three dimensional manifold which is invariant for inhomogeneous Lorentz transform. Since the carrier of