121. *On the Gibbs Phenomenon for* (K, 1) *Means*

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(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 13, 1965)

§ 1. Zygmund [9] introduced the following method of summability which is similar to the *Lebesgue method* $(R, 1)$[1]: When a series

$$(1) \qquad \sum_{n=0}^{\infty} u_n$$

is given, if

(i) the series

$$\frac{2}{\pi}\sum_{n=1}^{\infty} u_n \int_h^{\pi} \frac{\sin nt}{2\tan\frac{1}{2}t}\,dt$$

converges for small positive h, and if

(ii) the limit of

$$u_0+\frac{2}{\pi}\sum_{n=1}^{\infty} u_n \int_h^{\pi} \frac{\sin nt}{2\tan\frac{1}{2}t}\,dt$$

for $h\to+0$ exists and equals s. then he calls that the series (1) is *summable* $(K, 1)$ to s.

The convergence of (1) need not imply its summability $(K, 1)$ as well as in the case of the method $(R, 1)$. We shall study, in this note, the Gibbs phenomenon of the Fourier series

$$(2) \qquad \sum_{n=1}^{\infty} \frac{\sin nx}{n}$$

for the $(K, 1)$ means. Ching-Hsi Lee [5] proved the following

Theorem 1. *The* $(R, 1)$ *means of the series* (2) *does not present the Gibbs phenomenon at* $x=0$.

We shall prove here the following

Theorem 2. *The* $(K, 1)$ *means of the series* (2) *does not present the Gibbs phenomenon at* $x=0$.

Proof. Let

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1) We say that the series (1) is summable $(R, 1)$ to s, if $\sum_{n=1}^{\infty} u_n \frac{\sin nh}{nh}$ converges for small positive h, and if $\lim_{h\to+0}\left\{u_0+\sum_{n=1}^{\infty} u_n \frac{\sin nh}{nh}\right\}=s$. See, e.g., Hardy [1], p. 89, Zeller [8], p. 158.