118. Some Applications of the Functional Representations of Normal Operators in Hilbert Spaces. XVI

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Let $\{\lambda_\nu\}_{\nu=1,2,3,\dots}$, $D_j(j=1, 2, 3, \cdots, n)$, and $T(\lambda)$ be the same notations as those defined in Part XIII [cf. Proc. Japan Acad., Vol. 40, No. 7, 492–497 (1964)]; let $\chi(\lambda)$ be the sum of the first and second principal parts of $T(\lambda)$; and let us suppose that $\{\lambda_\nu\}$ is everywhere dense on a (closed or open) rectifiable Jordan curve Γ and that for any small positive ε the circle $|\lambda|=\sup_\nu|\lambda_\nu|+\varepsilon$ contains the mutually disjoint sets Γ, $D_1, D_2, \cdots, D_{n-1}$, and D_n inside itself. In this paper we shall discuss the respective behaviours concerning ρ of the maximum moduli of $\chi(\lambda)$ and $T(\lambda)$ on the circle $|\lambda|=\rho$ with $\sup_\nu|\lambda_\nu|<\rho<\infty$.

Theorem 43. Let $T(\lambda)$ be the function with singularities $\overline{\{\lambda_\nu\}}\cup\left[\bigcup_{j=1}^{n} D_j\right]$ stated above; let $\chi(\lambda)$ be the sum of the first and second principal parts of $T(\lambda)$; let $\sigma=\sup_\nu|\lambda_\nu|$; and let $M_\chi(\rho)$ denote the maximum modulus of $\chi(\lambda)$ on the circle $|\lambda|=\rho$ with $\sigma<\rho<\infty$. Then

$$M_\chi(\rho')\leqq M_\chi(\rho) \qquad (\sigma<\rho<\rho'<\infty),$$
$$M_\chi(\rho)\to\infty \qquad (\rho\to\sigma),$$

and for any ρ with $\sigma<\rho<\infty$

$$\text{(A)}\quad \frac{1}{2}\sqrt{\sum_{\mu=1}^{\infty}|a_\mu(\rho)+ib_\mu(\rho)|^2}\leqq M_\chi(\rho)\leqq\frac{1}{2}\sum_{\mu=1}^{\infty}|a_\mu(\rho)+ib_\mu(\rho)|<\infty,$$

where

$$\left.\begin{aligned} a_\mu(\rho)&=\frac{1}{\pi}\int_0^{2\pi}T(\rho e^{it})\cos\mu t\,dt\\ b_\mu(\rho)&=\frac{1}{\pi}\int_0^{2\pi}T(\rho e^{it})\sin\mu t\,dt\end{aligned}\right\}(\sigma<\rho<\infty,\ \mu=1, 2, 3, \cdots).$$

Proof. Let C denote the positively oriented circle $|\lambda|=\rho$ with $\sigma<\rho<\infty$, and let $R(\lambda)$ be the ordinary part of $T(\lambda)$. Then, as already demonstrated in Theorem 30 of Part XIII quoted above,

$$\frac{1}{2\pi i}\int_C\frac{T(\lambda)}{(\lambda-z)^k}d\lambda=\begin{cases}R^{(k-1)}(z)/(k-1)! & \text{(for every } z \text{ inside } C)\\ -\chi^{(k-1)}(z)/(k-1)! & \text{(for every } z \text{ outside } C),\end{cases}$$

where $k=1, 2, 3, \cdots$. Furthermore, as can be seen from the method of the proof of (5) [cf. Proc. Japan Acad., Vol. 38, No. 8, 452–456 (1962)], it is verified with the help of these relations that