153. Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. XVII

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Let $T(\lambda)$ be the function treated in Theorems 43, 44, and 45 of the preceding paper. Namely $T(\lambda)$ has as its singularity every point belonging to the bounded set $\overline{\{\lambda_\nu\}} \cup \left[\bigcup_{j=1}^{n} D_j\right]$ where the denumerably infinite set $\{\lambda_\nu\}$ is everywhere dense on a closed or an open rectifiable Jordan curve Γ and satisfies the condition that for any small positive ε the circle $|\lambda| = \sup_\nu |\lambda_\nu| + \varepsilon$ contains the mutually disjoint closed sets $\Gamma, D_1, D_2, \cdots, D_{n-1}$, and D_n inside itself [cf. Proc. Japan Acad., 40 (7), 492–497 (1964)]. In this paper we are mainly concerned with the distribution of c-points of the sum of the first and second principal parts of $T(\lambda)$ in the domain $\{\lambda: \sup_\nu |\lambda_\nu| < |\lambda| < \infty\}$, on the assumption that c is an arbitrary finite complex number.

Theorem 46. Let $\chi(\lambda)$ be the sum of the first and second principal parts of the above-mentioned function $T(\lambda)$; let $\sigma = \sup_\nu |\lambda_\nu|$; let c be an arbitrarily given finite non-zero complex number; let $n(\rho, c)$, $(\sigma < \rho < \infty)$, be the number of all the c-points, with due count of multiplicity, of $\chi(\lambda)$ in the domain $\Delta_\rho\{\lambda: \rho < |\lambda| < \infty\}$; let

$$N(\rho, c) = \int_\rho^\infty \frac{n(r, c)}{r} dr \qquad (\sigma < \rho < \infty);$$

and let

$$m(\rho, c) = \frac{1}{2\pi} \int_0^{2\pi} \log \frac{1}{[\chi(\rho e^{-it}), c]} dt \qquad (\sigma < \rho \leqq \infty),$$

where

$$[\chi(\rho e^{-it}), c] = \frac{|\chi(\rho e^{-it}) - c|}{\sqrt{(1 + |\chi(\rho e^{-it})|^2)(1 + |c|^2)}}.$$

Then the equality

$$N(\rho, c) + m(\rho, c) - m(\infty, c) = \frac{1}{2\pi} \int_0^{2\pi} \log \sqrt{1 + |\chi(\rho e^{-it})|^2}\, dt$$

holds for every ρ with $\sigma < \rho < \infty$; and in addition, $N(\rho, c)$, $m(\rho, c) - m(\infty, c)$, and the right-hand definite integral tend to 0 as ρ becomes infinite.

Proof. If we now consider the function $f(\lambda) \equiv \chi\left(\frac{\rho^2}{\lambda}\right)$, $(\sigma < \rho < \infty)$, of a complex variable λ, then $f(\lambda)$ is regular in the domain $D\{\lambda: 0 \leqq$