148. *Semigroups with a Maximal Homomorphic Image having Zero*[*)]

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Let S be a semigroup and suppose that S is homomorphic onto a semigroup S' with zero. Then S' is called a Z-homomorphic image of S. A Z-homomorphic image S_0 of S is called a maximal Z-homomorphic image of S if any Z-homomorphic image S' of S is a homomorphic image of S_0. An ideal T of a semigroup S is called a minimal ideal if it does not properly contain an ideal of S. Of course, a mininal ideal is unique if it exists. If S has a minimal ideal, S has a maximal Z-homomorphic image, but this converse is not true as Example 2 shows. This paper gives a necessary and sufficient condition for a semigroup to have a maximal Z-homomorphic image.

Let I be an ideal of a semigroup S. Then S/I denotes the Rees factor semigroup. The following lemmas are fundamental (cf. [1]).

Lemma 1. *Let I_1 and I_2 be ideals. If $I_1 \subseteq I_2$ then S/I_1 is homomorphic onto S/I_2.*

Lemma 2. *Let S' be any Z-homomorphic image of a semigroup S. Then there exists an ideal I of S such that S/I is homomorphic onto S'.*

Lemma 3. *If $I_2 \subset I_1$ and if S/I_1 is homomorphic onto S/I_2 then there is an ideal I_3 of S such that $I_1 \subset I_3$ and S/I_3 is homomorphic onto S/I_1.*

Let $\mathfrak{S}$ be the family of all ideals of a semigroup S. Hereafter, we shall call a subfamily of $\mathfrak{S}$ a family of ideals.

Theorem 1. *A semigroup S has a maximal Z-homomorphic image if and only if there is a non-empty family $\mathfrak{F}$ of ideals such that the following conditions are satisfied.*

(1.1) *If $I_\xi \in \mathfrak{F}$ and $I_\eta \in \mathfrak{S}$ such that $I_\eta \subseteq I_\xi$ then $I_\eta \in \mathfrak{F}$.*

(1.2) *If $I_\xi, I_\eta \in \mathfrak{F}$, and $I_\eta \subseteq I_\xi$, then S/I_ξ is homomorphic onto S/I_η.*

Proof. Necessity of (1.1) and (1.2): Suppose that S has a maximal Z-homomorphic image S_0. By Lemma 2, we may assume that $S_0 = S/I_0$ where I_0 is an ideal of S. $\mathfrak{F}$ is defined to be the system

*) The abstract of this paper was partly announced in [3] by one of the authors.