# 143. On Axiom Systems of Propositional Calculi. V

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In the first note of this series, Y. Imai and K. Iséki proved that Hilbert axioms of two valued propositional calculus imply (F), (R), ($L_1$), ($L_2$), ($L_3$), ($S_1$), and ($S_2$) axioms systems (For notations and rules of inference, see [3]). In the second note [2], Y. Arai proved that the ($L_1$)-system is equivalent to a modification of ($L_1$)-system. In our seminar, he showed that axioms of the system are independent each other. In a later paper, we shall publish the result with other results on independences of axioms. In the third note (2), Y. Arai published deductions from the ($L_3$)-system to all other axiom systems. In his note [4], K. Iséki proved that an axiom system given by E. Mendelson implies all other systems of axioms mentioned in [3].

In this note, we shall prove that the Russell axioms of propositional calculus imply Lukasiewicz ($L_1$)-system. In the next note, S. Tanaka, one of the present authors, proves that (R)-system implies all other systems of axioms of propositional calculus given in [3] and [4]. As already said, we use only two rules of inference, i.e. rules of substitution and detachment.

1 *CpCqp*,

2 *CCpqCCqrCpr*,

3 *CCpCqrCqCpr*,

4 *CNNpp*,

5 *CCpNpNp*,

6 *CCpNqCqNp*.

These six theses are an axiom system given by B. Russell. We shall prove the following theses from these axioms:

3 *r*/*p* **C*1—7,

7 *CqCpp*.

7 *q*/*CpCqp* **C*1—8,

8 *Cpp*.

6 *p*/*Np*, *q*/*p* **C*8 *p*/*Np*—9,

9 *CpNNp*.

3 *p*/*Cpq*, *q*/*Cqr*, *r*/*Cpr* **C*2—10,

10 *CCqrCCpqCpr*.

10 *p*/*q*, *q*/*NNp*, *r*/*p* **C*4—11,

11 *CCqNNpCqp*.

2 *p*/*CNpNq*, *q*/*CqNNp*, *r*/*Cqp* **C*6 *p*/*Np*—*C*11—12,

12 *CCNpNqCqp*.