2. *Remarks on Periodic Solutions of Linear Parabolic Differential Equations of the Second Order*

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1. **Introduction.** Let E^m be the m-dimensional Euclidian space of points $x=(x_1, \cdots, x_m)$ and let Ω be an unbounded domain in E^m with boundary $\partial\Omega$. We set $Q=\{(x, t): x \in \Omega, -\infty<t<\infty\}$ and $\partial Q=\{(x, t): x \in \partial\Omega, -\infty<t<\infty\}$. Q is an infinite cylinder in E^{m+1} whose base is Ω and whose (lateral) boundary is ∂Q. $\bar{Q}$ denotes the closure of Q.

In this note we shall be concerned with periodic solutions of the first boundary problem in Q for linear second order parabolic equations having periodic coefficients and right members.[1)]

We shall briefly discuss the existence and the uniqueness of the periodic solutions which may grow exponentially as the variable x tends to infinity.

In our discussion we shall use the method similar to that employed by M. Krzyżański in regard to elliptic and parabolic boundary problems in unbounded domains [1–3].

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2. Let us consider the equation.

$$(1)\qquad Lu=\sum_{i,j=1}^{m} a_{ij}(x, t)\frac{\partial^2 u}{\partial x_i \partial x_j}+\sum_{i=1}^{m} b_i(x, t)\frac{\partial u}{\partial x_i}+c(x, t)u-\frac{\partial u}{\partial t}=f(x, t) \qquad \text{in } Q,$$

and the boundary condition

$$(2)\qquad u(x, t)=\varphi(x, t) \qquad \text{on } \partial Q.$$

We shall need the following assumptions:

1°. The functions $a_{ij}, b_i, c, f,$ and φ are continuous in $\bar{Q}$ and periodic with period T $(T>0)$.

2°. There exist positive constants $A, B,$ and C such that

$$|a_{ij}| \leqq A, \ |b_i| \leqq B, \ c \leqq -C \qquad \text{in } \bar{Q}.$$

3°. The form $\sum_{i,j=1}^{m} a_{ij}\xi_i\xi_j$ is positive definite in $\bar{Q}$.

Definition. We shall say that a function $w(x, t)$ belongs to class $\bar{E}_1(K)$ ($\underline{E}_1(K)$) if there exist positive constants M_0 and k_0 $(0<k_0<K)$ such that

1) Here and throughout by a *periodic* function is meant one which is periodic in the time variable t.