107. Γ-Bundles and Almost Γ-Structures

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In [2]-II, the author states that if X is a normal paracompact topological space, then we can define a sheaf of groups $H_*(n)_c$ over X and there is a 1 to 1 correspondence between the set of equivalence classes of n-dimensional topological microbundles over X and $H^1(X, H_*(n)_c)$. In this note, first we give the precise definition of $H_*(n)_c$ and (topological) connection of topological microbundles. Next, using $H_*(n)_c$, we define the almost Γ-structure if X is a topological manifold and give an integrability condition of almost Γ-structures.

1. *Definition of the sheaf $H_*(n)_c$.* We denote the semigroup of all homeomorphisms of $\boldsymbol{R}^n$ into $\boldsymbol{R}^n$ which fix the origin by $E_0(n)$. $E_0(n)$ is regarded to be a topological semigroup by compact open topology. We denote by X a topological space with $\{U_\alpha(x)\}$ the neighborhood basis of x. The semigroup of all continuous maps from $U_\alpha(x)$ into $E_0(n)$ is denoted by $H(U_\alpha(x), E_0(n))$. For $f \in H(U_\alpha(x), E_0(n))$, we set

$$f(y, a)=(y, f(y)(a)).$$

By definition, f is a homeomorphism from $U_\alpha(x)\times \boldsymbol{R}^n$ into $U_\alpha(x)\times \boldsymbol{R}^n$.

Definition. We call f and g are equivalent if f and g coincide on some neighborhood of $x\times 0$ in $U_\alpha(x)\times \boldsymbol{R}^n$.

The set of equivalence classes of $H(U_\alpha(x), E_0(n))$ by this relation is denoted by $H_*(U_\alpha(x), E_0(n))$.

If $U_\alpha(x)$ contains $U_\beta(x)$, then there is a homeomorphism $\bar{r}^\alpha_\beta$: $H_*(U_\alpha(x), E_0(n))\rightarrow H_*(U_\beta(x), E_0(n))$ induced from the restriction homeomorphism. We set

$$(1) \qquad H_*(n)_x=\lim\, [H_*(U_\alpha(x), E_0(n)), \bar{r}^\alpha_\beta].$$

Lemma 1. *$H_*(n)_x$ is a group.*

If $f\in H(U, E_0(n))$, then its class in $H_*(n)_x$ is denoted by f_x. We set

(2) $U(f_x, V(x))=\{f_y \mid y\in V(x)\}$, $V(x)$ is a neighborhood of x in X. In $\bigcup_{x\in X}H_*(n)_x$, we take $\{U(f_x, V(x))\}$ to be the neighborhood basis of f_x, then $\bigcup_{x\in X}H_*(n)_x$ becomes a sheaf of groups over X. We denote this sheaf by $H_*(n)_c$.

2. *The cohomology set $H^1(X, H_*(n)_c)$.* **Theorem 1.** *If X is a normal paracompact topological space, then there is a* 1 *to* 1