5. *Some Generalizations of V. Trnkova's Theorem on Unions of Strongly Paracompact Spaces*

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V. Trnkova [5] has recently investigated the unions of strongly paracompact spaces and he has proved the following interesting theorem:

If space $X=X_1\cup X_2$, X_1, X_2 are closed and strongly paracompact subspaces, and the space $X_1\cap X_2$ has the locally Lindelöf property, then X is itself strongly paracompact. In this note, we shall obtain some generalizations of V. Trnkova's Theorem.

Let us quickly recall the definitions of terms which are used in this note. Let X be a topological space, and $\mathfrak{N}$ be a collection of subsets of X. The collection $\mathfrak{N}$ is said to be *locally finite* if every point of X has a neighborhood which intersects only finitely many elements of $\mathfrak{N}$. The collection $\mathfrak{N}$ is said to be *star finite* (resp. *star countable*) if each element of $\mathfrak{N}$ intersects only finitely (resp. only countably) many elements of $\mathfrak{N}$. Finally, X is said to be *paracompact* (resp. *strongly paracompact*) if X is Hausdorff and every open covering of X has a locally finite open covering (resp. star finite open covering) of X as a refinement.

§1. Generalizations. In this section, we shall get some generalizations of V. Trnkova's Theorem. At first, we shall show some lemmas.

Lemma 1. *Let $\mathfrak{B}=\{B_\alpha\mid\alpha\in A\}$ be a locally finite closed covering of a regular space X. If each B_α has the locally Lindelöf property as a subspace, then X has the locally Lindelöf property.*

Proof. Let x_0 be an arbitrary point of X. Then, there exists a closed neighborhood $V_0(x_0)$ of x_0 in X such that $V_0(x_0)$ intersects only all the members $B_{\alpha_1},\cdots,B_{\alpha_n}$ containing x_0. For each $i=1,2,\cdots,n$, by the locally Lindelöf property of B_{α_i}, we have the closed neighborhood $V_i(x_0)$ of x_0 in X such that $V_i(x_0)\cap B_{\alpha_i}$ has the Lindelöf property. Let $V=\bigcap_{i=0}^{n} V_i(x_0)$, then V is a neighborhood of x_0 and $V=V\cap(\bigcup_{i=1}^{n} B_{\alpha_i})$ $=\bigcup_{i=1}^{n}(V\cap B_{\alpha_i})$. This relation implies the Lindelöf property of V. Thus we get Lemma 1.

Lemma 2. *Let $\{F'_\alpha\mid\alpha\in A\}$ be a locally finite closed covering of a regular space X where the index set A is a well ordered set. If we define as follows: $F_1=F'_1$, $F_\alpha=\overline{F'_\alpha-\bigcup_{\beta<\alpha}F'_\beta}$ for each $\alpha>1$, then*