83. *A Criterion for the Separable Axiomatization of Gödel's S_n*

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This report is an extension to our papers [2] and [3]. And we use notations and results of them without mentioning.

In this paper, we report a criterion for an axiom scheme to give a separable axiomatic system for S_n by adding it to Dummett's LC, and we also report that there is no intermediate axiomatic system between S_n and S_{n+1}. Our result is also an extension to that obtained by Hanazawa [1] in the following form, though we do not suppose familiarity with it.

Theorem 1 (*By Hanazawa*). *The system $LI+A$ is equivalent to the usual classical system S_1 if and only if A is valid in S_1 but not in S_2.*

Since axiomatic systems are known for S_n's, the validity in S_n is equivalent to the provability in S_n. Though Hanazawa does not mention explicitly, the above theorem implies the following

Corollary 2. *There is not an intermediate axiomatic system between S_1 and S_2.*

Further, we have a stronger

Corollary 3. *If $S_1 \supsetneqq L \subset LI$, then necessarily $S_2 \supset L$.*

Proof. Suppose that $L \vdash A$ but not that $S_2 \vdash A$. Then $S_1 \supset\subset LI+A$ since $S_1 \vdash A$. On the other hand, we obviously have that $L \supset LI+A$. This is contrary to our hypothesis.

Before we mention our theorem, we remark that the above theorem does not generally hold for S_n and S_{n+1} in the above form. Let us take the formula P_n of Nagata, for example. We know that P_n is valid in S_n but not in S_{n+1}, but as we reported in [2], $LI+P_n$ is not equivalent to S_n. So we prove a similar theorem in the following form.

Theorem 4. *$LC+A \supset\subset S_n$ if and only if $S_n \vdash A$ and not $S_{n+1} \vdash A$.*

Before we prove the theorem, we quote some lemmas from our previous papers without proof.

Lemma 5. Suppose a formula A has k distinct propositional variables at the most. *Then $LC \vdash A$ if and only if $S_{k+1} \vdash A$.*

Lemma 6. Suppose that A does not contain the logical opera-