97. The Asymptotic Formula for the Trace of Green Operators of Elliptic Operators on Compact Manifold*)

By Daisuke FUJIWARA

Department of Mathematics, University of Tokyo

(Comm. by Zyoiti SUETUNA, M.J.A., June 12, 1967)

§1. **Preliminaries.** It is interesting to know the asymptotic expansion of the trace of the Green operator $(P+\tau)^{-1}$ of elliptic pseudo-defferential operator P operating on sections of a complex vector bundle X over a compact differentiable manifold M. The asymptotic expansion is obtained by introducing a special class of pseudo-differential operators on $X\otimes 1_{R^1}$, where 1_{R^1} is the trivial line bundle over the real line $\boldsymbol{R}^1$. This is called β-pseudo-differential operator for the time being. Proofs are omitted, but will be published elsewhere.

In the following, we shall follow the usual notations in [1] or [2] for the special spaces of distributions.

The author expresses his hearty thanks to Professor S. Itô who has kindly read through this manuscript with criticism.

§2. **β-pseudo-differential operators.** Consider a σ-compact differentiable n-manifold M and a smooth complex vector bundle X of dimension l over M. Let Y be the bundle over $M\times\boldsymbol{R}^1$ induced from X by the projection $M\times\boldsymbol{R}^1\to M$. We identify the bundle Y with $X\otimes 1_{R^1}$. We denote the generic point of $M\times\boldsymbol{R}^1$ by (x, s). When Z is a vector bundle over a differentiable manifold N, we denote the space of C^∞ sections (resp. C^∞ sections with compact support) over an open subset U of N by $\mathcal{E}(U, Z)$ (resp. $\mathcal{D}(U, Z)$). In the following, we denote the annulus $\{(\rho, \sigma)\in\boldsymbol{R}^2 : \frac{1}{2}\leq\rho^2+\sigma^2\leq 2\}$ by A.

Definition 1. A continuous linear map P from $\mathcal{D}(M, X)\widehat{\otimes}\mathcal{S}'(\boldsymbol{R}^1)$ into $\mathcal{E}(M, X)\widehat{\otimes}\mathcal{S}'(R^1)$ is called a β-pseudo-differential operator of order s_0, if there is a sequence $s_0>s_1>\cdots\to-\infty$ of reals such that, for all $f\in\mathcal{D}(M, X)$ and $g\in\mathcal{E}(M)$ which is real valued with $dg\neq 0$ on supp f, $e^{-i\lambda(g\rho+s\sigma)}P(fe^{i\lambda(g\rho+s\sigma)})$ is the pull back of a section $p(f, g\rho, x, \sigma)$ of X and there holds the asymptotic expansion

$$p(f, g\rho, x, \sigma)\sim\sum_0^\infty p_j(f, g\rho, x, \sigma)\lambda^{s_j}, \qquad \lambda\to\infty, \tag{2.1}$$

which has the following property: For any integer $N>0$ and com-

*) This research was partly supported by The Sakkokai Foundation.