164. *On Equivalences of Laws in Elementary Protothetics. I*

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In his paper [1], J. Słupecki has given some generalizations of the six laws that have described the properties of functions of one argument in elementary protothetics.

In this paper, by using the well known rules of inference and substitution we shall show that each laws on functions of one argument is equivalent to its corresponding laws of functions of two arguments. J. Słupecki has not given the proofs of the equivalences given below in his parper [1]. The rules of inference and of substitution used in the systems of elementary protothetics has given in J. Słupecki [1] in detail.

First of all, we shall prove the equivalence of the theorems (a) and (a′) which have been called the *law of development*:

(a) $[f, p]\{f(p)\equiv(f(1)\cdot p\vee f(0)\cdot\sim(p))\}$,

(a′) $[f, p, q]\{f(p, q)\equiv(f(1, 1)\cdot p\cdot q\vee f(1, 0)\cdot p\cdot\sim(q)\vee f(0, 1)\cdot\sim(p)\cdot q\vee f(0, 0)\cdot\sim(p)\cdot\sim(q))\}$.

Then we shall first prove the following theorem to show the equivalence mentioned above.

Theorem 1. $[f, \mathrm{p}, q]\{[f, r]\{f(r)\supset(f(1)\cdot r\vee f(0)\cdot\sim(r))\}\cdot f(p, q)\supset(f(1, 1)\cdot p\cdot \mathrm{q}\vee f(1, 0)\cdot p\cdot\sim(q)\vee f(0, 1)\cdot\sim(p)\cdot q\vee f(0,0)\cdot\sim(p)\cdot\sim(q))\}$.

Proof. The idea of the proof is due to J. Słupecki ([1], p. 71).

(1) $[f, r]\{f(r)\supset(f(1)\cdot r\vee f(0)\cdot\sim(r))\}$,

(2) $f(p, q)\supset$

D1 $[f, p, q]\{\psi\langle f, p\rangle(q)\equiv f(q, p)\}$.

By replacing a functor $\psi\langle f, q\rangle$ for the functor f, a variable p for the variable r in the expression (1), we obtain the following expression.

(3) $\psi\langle f, q\rangle(p)\supset(\psi\langle f, q\rangle(1)\cdot p\vee\psi\langle f, q\rangle(0)\cdot\sim(p))$, (D1; 1)

(4) $\psi\langle f, q\rangle(p)$, (D1; 2)

we obtain the following expression by applying the rule of detachment from (3) and (4).

(5) $\psi\langle f, q\rangle(1)\cdot p\vee\psi\langle f, q\rangle(0)\cdot\sim(p)$, (3; 4)

(6) $f(1, q)\cdot p\vee f(0, q)\cdot\sim(p)$, (D1; 5)

(7) $\chi\langle f, 1\rangle(q)\equiv(\chi\langle f, 1\rangle(1)\cdot q\vee\chi\langle f, 1\rangle(0)\cdot\sim(q))$. (a)

To obtain (7) we have used the functor obtained by the definition D2 given below:

D2 $[\mathrm{f}, p, q]\{\chi\langle f, p\rangle(q)\equiv f(p, q)\}$,