159. *On Many-Valued Lukasiewicz Algebras*

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(Comm. by Kinjirô KUNUGI, M.J.A., Oct. 12, 1967)

The many-valued Lukasiewicz algebras were introduced by Prof. Gr. C. Moisil [2] as models for J. Lukasiewicz' many-valued propositional calculus.

In this note, we give a system with a smaller number of axioms for the many-valued Lukasiewicz algebras. Also is considered a generalization for the notion of strict chrysippien element and we give conditions in which a many-valued Lukasiewicz algebra is a Kleene algebra.

1. The many-valued Lukasiewicz algebra is a lattice L which satisfies [2] the following axioms:

$L1$) L is a distributive lattice with first and last element.

$L2$) L has an involutive duality named negation:

$$N(x \cap y) = Nx \cup Ny,$$
$$N(x \cup y) = Nx \cap Ny,$$
$$NNx = x.$$

$L3$) L has $n-1$ endomorphisms $\sigma_1, \cdots, \sigma_{n-1}$:

$$\sigma_i(x \cup y) = \sigma_i x \cup \sigma_i y,$$
$$\sigma_i(x \cap y) = \sigma_i x \cap \sigma_i y.$$

$L4$) The elements $\sigma_i x$ are chrysippien

$$\sigma_i x \cup N\sigma_i x = 1,$$
$$\sigma_i x \cap N\sigma_i x = 0.$$

$L5$) The elements $\sigma_i x$ form a linear ordered lattice

$$\sigma_1 x \subset \sigma_2 x \subset \cdots \subset \sigma_{n-1} x.$$

$L6$) There is the relation

$$\sigma_i \sigma_j x = \sigma_j x \quad \text{for any } i, j.$$

$L7$) There is the relation

$$\sigma_i Nx = N\sigma_j x, \quad \text{where } j = \varphi(i).$$

$L8$) If $\sigma_i x = \sigma_i y$ $(i=1, \cdots, n-1)$ then $x = y$.

$L8$ is called the determination principle.

Prof. Gr. C. Moisil proves [3] that

$$\sigma_i 0 = 0, \quad \sigma_i 1 = 1 \quad (1 \leq i \leq n-1). \tag{1}$$

If we introduce [2] the Lagrange's fonctions

$$\lambda_i x = \sigma_{n-i} x \cap N\sigma_{n-i-1} x \qquad (0 \leq i \leq n-1), \tag{2}$$

where $\sigma_0 x = 0$, $\sigma_n x = 1$, then [3] the following formulas hold:

$$\sigma_{n-i} x = \lambda_i x \cup \lambda_{i+1} x \cup \cdots \cup \lambda_{n-1} x, \tag{3}$$