15. Axiom Systems of Aristotle Traditional Logic. III

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In this paper, we shall give a new axiom system of Aristotle traditional logic. Some axiom systems have been obtained by J. Lukasiewicz [4], [5], I. Bochenski [1], N. Kretzmann [3], K. Iséki [2], and S. Tanaka [6]. K. Iséki has given a method to find its axiom systems. For the detail, see [2]. In this paper, we use the following notations. For the categorical sentences,

- 1) Aab: Every a is b,
- 2) Iab: At least one a is b,
- 3) Oab: At least one a is not b,
- 4) Eab: No a is b.

For the functors,

- 1) C: implication, 2) N: negation, 3) K: conjunction. Then we have
 - $D1. \quad Eab = NIab,$
 - $D2. \quad Oab = NAab.$

For the moods and figures,

- 1) XY_1 : CXabYab, 2) XY_2 : CXabYba,
- 3) XYZ_1 : CKXabYcaZcb, 4) XYZ_2 : CKXabYcbZca,
- 5) XYZ_3 : CKXabYacZcb,
- 6) XYZ₄: CKXabYbcZca.

Under these notations, the Lukasiewicz axiom system is written as follows:

- L1. Aaa,
- L2. Iaa,
- $L3. AAA_1.$
- $L4. AII_3.$

The following deduction rules T1, T2, T3 from the classical propositional calculus are used in our discussion.

- T1. $CK\alpha\beta\gamma\rightarrow CK\beta\alpha\gamma$,
- $T2. CK\alpha\beta\gamma, C\gamma\delta \rightarrow CK\alpha\beta\delta,$
- T3. $C\alpha\beta \rightarrow CN\beta N\alpha$.

For the simplicity, we shall write these as

- T1. $\alpha\beta\gamma\rightarrow\beta\alpha\gamma$,
- $T2. \quad \alpha\beta\gamma + \gamma\delta \rightarrow \alpha\beta\delta$,
- T3. $\alpha\beta \rightarrow N\beta N\alpha$.

In our previous notes, K. Iséki and S. Tanaka have given some important