25. *Cohomology Operations in Iterated Loop Spaces*

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1. Introduction. In [3], Dyer and Lashof have determined the mod p homology structure of iterated loop spaces by use of extended p-th power operations, where p always denotes a prime. This operation is a generalization of H-squaring (for $p=2$) defined by Araki-Kudo [2], and operates on mod p homology group of H_p^∞-spaces X, especially iterated loop spaces. Let $Q_i^{(p)}: H_n(X; Z_p) \to H_{np+i}(X; Z_p)$ be Dyer-Lashof's extended powers. For odd p, we denote operations $Q_{(p)}^j: H_n(X; Z_p) \to H_{n+2j(p-1)}(X; Z_p), j=0, 1, \cdots,$ by $Q_{(p)}^j x = (-1)^{i+m(n^2+n)/2}(m!)^n Q_{(2j-n)(p-1)}^{(p)} x$, $x \in H_n(X; Z_p)$, $m=(p-1)/2$, and for $p=2$, $Q_{(2)}^j: H_n(X; Z_p) \to H_{n+j}(X; Z_p)$ by $Q_{(2)}^j x = Q_{j-n}^{(2)} x$.

The operation $Q_{(p)}^j$ has the following properties: 1. *$Q_{(p)}^j$ is a homomorphism; 2. For odd p, $Q_{(p)}^j x=0$ if* deg *$x>2j$ and $Q_{(p)}^j x = x^p$ if* deg *$x=2j$, and for $p=2$, $Q_{(2)}^j x=0$ if* deg *$x>j$ and $Q_{(2)}^j x = x^2$ if* deg *$x=j$;* 3. *$Q_{(p)}^j(x\cdot y) = \sum_{k+l=j} Q_{(p)}^k x \cdot Q_{(p)}^l y$;* 4. *$Q_{(p)}^j$ commutes with the suspension homomorphism σ associated with the fibering of the contractible total space, $\sigma Q_{(p)}^j = Q_{(p)}^j \sigma$.*

Our purpose is to determine the relation between $Q_{(p)}^j$ and the Steenrod reduced power operations ρ^n (squaring operations Sq^n for $p=2$). To state the results, we denote by ρ_*^n the dual operation of ρ^n, i.e., defined by

$$\langle \rho_*^n x, y\rangle = \langle x, \rho^n y\rangle \text{ for } x \in H_*(X; Z_p),\ y \in H^*(X; Z_p).$$

Let $\binom{a}{b}$ be the binomial coefficient with the following conversions: $\binom{a}{b}=0$ for a or $b<0$ and $\binom{a}{b}=1$ for $b=0, a\geq 0$. Δ denotes the homology Bockstein operation. Then we have

Main theorem. *For odd p,*

$$\rho_*^n Q_{(p)}^{n+s} = \sum_i (-1)^{n+i} \binom{s(p-1)}{n-pi} Q_{(p)}^{s+i} \rho_*^i,$$

$$\rho_*^n \Delta Q_{(p)}^{n+s} = \sum_i (-1)^{n+i} \binom{s(p-1)-1}{n-pi} \Delta Q_{(p)}^{s+i} \rho_*^i + \sum_i (-1)^{n+i+1} \binom{s(p-1)-1}{n-pi-1} Q_{(p)}^{s+i} \rho_*^i \Delta,$$

and for $p=2$

$$Sq_*^n Q_{(2)}^{n+s} = \sum_i \binom{s}{n-2i} Q_{(2)}^{s+i} Sq_*^i.$$