60. *On Some Mixed Problems for Fourth Order Hyperbolic Equations*

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(Comm. by Kinjirô KUNUGI, M. J. A., April 12, 1968)

§1. Introduction. We consider some mixed problems for fourth order hyperbolic equations. Let S be a smooth and compact hypersurface in R^n and Ω be the interior or exterior of S. Let

$$\text{(E)}\quad Lu=\left(\frac{\partial^4}{\partial t^4}+(a_1+a_2+a_3)\frac{\partial^2}{\partial t^2}+a_3a_1\right)u+B\left(x,\,t,\,\frac{\partial}{\partial t},\,D\right)u=f.$$

Here $a_k(k=1, 2, 3)$ are the following operators:

$$a_k=-\sum_{ij}^{n}\frac{\partial}{\partial x_i}\left(a_{k,ij}(x)\frac{\partial}{\partial x_j}\right)+b_k(x, D),$$

$$(1.1)\qquad a_{k,ij}(x)=a_{k,ji}(x)\ \text{are real,}$$

$$\sum_{ij}^{n}a_{k,ij}(x)\xi_i\xi_j\geq\delta|\xi|^2,\ (\delta>0)$$

$$\text{for every }(x,\xi)\in\Omega\times R^n\ (k=1, 2, 3),$$

B denotes an arbitrary third order differential operator and b_k are first order operators. Let us assume that all coefficients are sufficiently differentiable and bounded in $\bar{\Omega}$ or in $\bar{\Omega}\times(0,\infty)$.

Recently S. Mizohata [1] treated mixed problems for the equations of the form

$$L=\prod_{i=1}^{m}\left(\frac{\partial^2}{\partial t^2}+c_i(x)a(x, D)\right)+B_{2m-1},\quad c_i(x)>c_{i+1}(x),\quad c_i(x)>0$$

$$(i=1,\cdots,m).$$

Let us consider the case where $m=2$. The above equation has the form

$$\frac{\partial^4}{\partial t^4}+(c_1(x)+c_2(x))a\frac{\partial^2}{\partial t^2}+c_1c_2a^2+(\text{operator of third order}).$$

Now it is not difficult to see that this operator can be considered as a special class of (E), by putting $a_1=\alpha c_1a$, $a_2=(1-\alpha)c_1a+\left(1-\frac{1}{\alpha}\right)c_2a$, α being a constant less than 1 chosen closely to 1. We consider the case where the operators a_k have some relations only at the boundary. Let us denote the Sobolev space $H^p(\Omega)$ simply by H^p, and its norm by $\|\cdot\|_p$ and denote the closure of $\mathcal{D}(\Omega)$ in H^1 by $\mathcal{D}^1_{L^2}$. Define

$$D(a_k)=\{u\in H^3\cap\mathcal{D}^1_{L^2};\ a_ku\in\mathcal{D}^1_{L^2}\}.$$

Namely, $u\in H^3$ belongs to $D(a_k)$ means that not only u itself but also