52. *On Generalized Integrals. II*

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In the preceding paper [5], we proposed a question whether the set of $(E.R.)$ integrable functions can be obtained as a completion of the set $\mathcal{E}$ with respect to some reasonable topology and rank ($\mathcal{E}$ stands for the set of step functions on $[a, b]$). The aim of a series of these papers is to give a positive answer to it. To do this, first of all in the Note I we introduced on $\mathcal{E}$ a topology and a rank so that $\mathcal{E}$ should become a ranked space. We proved that, when $u:\{V_n(f_n)\}$ is a fundamental sequence in $\mathcal{E}$, $f_n(x)$ converges to a finite function $f(x)$ a.e. and $\int_a^b f_n(x)dx$ converges to a finite limit, that is, every fundamental sequence u determines a function $J(u)=f(x)$ and a value $I(v)=\lim_{n\to\infty}\int_a^b f_n(x)dx$. Moreover, in this paper, we will establish that when we agree with two functions equal if they differ only in a set of measure zero, each maximal collection u^* in $\mathcal{E}$ determines a function which we can associate to this u^*. We denote this function by $J(u^*)$. Let us denote, by $\boldsymbol{K}$, the set of those functions $f(x)$ for which there exist fundamental sequences u with $J(u)=f(x)$, and denote, by $\boldsymbol{U}$, the set of all maximal collections. Then, *$J(u^*)$ is a $(1, 1)$ mapping of $\boldsymbol{U}$ onto $\boldsymbol{K}$* (Theorem 1). Furthermore, $\boldsymbol{K}$ coincides with the set of $(E.R.)$ integrable functions in the special sense (or A-integrable functions). It results from I, Corollary 2)[1] that for $u \in u^*$ and $v \in u^*$, we have $I(u)=I(v)$. Therefore, we can write this value $I=I(u^*)$. We take $I(f)=I(J^{-1}(f))$ as the value of the integral of $f(x)$ belonging to $\boldsymbol{K}$. Theorem 2 shows that $I(f)=(A)\int_a^b f(x)dx=(E.R.)\int_a^b f(x)dx$ for all $f \in \boldsymbol{K}$.

3. The mapping $J(u^*)$. Let us remark that in the ranked space $\mathcal{E}$ defined in the Note I, the *fundamental sequence* is defined in the following form: a monotone decreasing sequence of neighbourhoods $\{V_n(f_n)\,;\, n=0, 1, 2, \cdots\}$ with $V_n(f_n) \in \mathfrak{B}_{\nu_n}$ is said to be fundamental if there exists a sub-sequence $\{V_{n_i}(f_{n_i})\,;\, i=0, 1, 2, \cdots\}$ such that $f_{n_{2i}}=f_{n_{2i+1}}$ and $\nu_{n_{2i}}<\nu_{n_{2i+1}}$ (without the equality).

We continue the study of the fundamental sequence in $\mathcal{E}$. First, we show a few Lemmas.

1) The reference number indicates the number of the Note.