# 49. *Calculus in Ranked Vector Spaces. II*

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**1.6. Ranked vector space.** In what follows we denote by $\mathfrak{R}$ the space of all real numbers with the usual topology.

(1.6.1) **Definition.** A space $E$ which is satisfying the following Conditions (I), (II) is called a *ranked vector space.*

(1.6.2) (I) $E$ is a vector space over the real or complex numbers and there is a countably family $\mathfrak{V}_0(0), \mathfrak{V}_1(0), \mathfrak{V}_2(0), \cdots, \mathfrak{V}_n(0), \cdots$ where each $\mathfrak{V}_n(0)$ consists of subsets of $E$. Let $\mathfrak{V}(0) = \cup \mathfrak{V}_n(0)$, then it satisfies the following conditions:

(A) Every $V$ belonging to $\mathfrak{V}(0)$ contains zero;

(B) For any $U, V \in \mathfrak{V}(0)$, there exists a $W \in \mathfrak{V}(0)$ such that
$$W \subset U \cap V;$$

(a) For any $U \in \mathfrak{V}(0)$, and for an integer $n$ $(0 \leq n < \omega_0)$, there exists an integer $m$ and a $V \in \mathfrak{V}(0)$ such that
$$m \geq n, \quad V \in \mathfrak{V}_m(0), \quad \text{and} \quad V \subset U;$$

(b) $E \in \mathfrak{V}_0(0)$.

With each element $x \in E$ there is associated a non-empty set $\mathfrak{V}(x)$ as follows:
$$\mathfrak{V}(x) = \{x + V;\ V \in \mathfrak{V}(0)\}.$$
Every element $U = x + V \in \mathfrak{V}(x)$ is called a *neighborhood* of a point $x$. Further, there is a countably system $\{\mathfrak{V}_n\}$ defined by
$$\mathfrak{V}_n = \{x + V;\ x \in E,\ V \in \mathfrak{V}_n(0)\},$$
for $n = 0, 1, 2, \cdots$.

(1.6.3) (II) In $E$ the following axioms hold [1]:

(1) *There exists a non-negative function $\phi(\lambda, \mu)$, defined for $\lambda \geq 0$ and $\mu \geq 0$, such that $\lim_{\lambda, \mu \to \infty} \phi(\lambda, \mu) = \infty$, and the following holds: if $U \in \mathfrak{V}_l(0)$, $V \in \mathfrak{V}_m(0)$, $W \in \mathfrak{V}_n(0)$, $n \leq \phi(l, m)$, and $U + V \subset W$, then there is an integer $n^* \geq \phi(l, m)$, and a neighborhood $W^* \in \mathfrak{V}_{n^*}(0)$, such that*
$$U + V \subset W^* \subset W.$$

(2) *There exists a non-negative function $\psi(\lambda, \mu)$ defined for $\lambda \geq 0$ and $\mu \geq 1$ such that $\lim_{\lambda \to \infty} \psi(\lambda, \mu) = \infty$, for each fixed $\mu$, and the following holds: let $\alpha$ be a scalar with $|\alpha| \geq 1$. If $U \in \mathfrak{V}_m(0)$, $V \in \mathfrak{V}_n(0)$, $\alpha U \subset V$, and $n \leq \psi(m, |\alpha|)$, then there is an integer $n^* \geq \psi(m, |\alpha|)$ and a $V \in \mathfrak{V}_{n^*}(0)$ such that*
$$\alpha U \subset V^* \subset V.$$