48. *Calculus in Ranked Vector Spaces. I*

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§ 1. Ranked vector space. 1.1. Ranked space. Let E be a neighborhood space, i.e., with every element $x \in E$ there is associated a non-empty set $\mathfrak{V}(x)=\{V(x)\}$ of subsets of E such that

(1.1.1) (1) $\quad V(x) \in \mathfrak{V}(x) \Rightarrow V(x) \ni x$;

(2) For any $U(x)$, $V(x) \in \mathfrak{V}(x)$, there exists a $W(x) \in \mathfrak{V}(x)$ such that

$$W(x) \subset U(x) \cap V(x);$$

(3) $$E \in \mathfrak{V}(x).$$

Every element $V(x)$ of $\mathfrak{V}(x)$ is called a *neighborhood* of a point $x \in E$.

(1.1.2) **Definition.** A neighborhood space E, on which a countably system $\mathfrak{V}_0, \mathfrak{V}_1, \mathfrak{V}_2, \cdots, \mathfrak{V}_n, \cdots$ consisting of neighborhoods ($E \in \mathfrak{V}_0$) is defined, is called a ranked space with the indicator ω_0 if and only if for every $x \in E$, $U(x) \in \mathfrak{V}(x)$ and for an integer n $(0 \leq n < \omega_0)$ there exists an integer m $(0 \leq m < \omega_0)$ and a neighborhood $V(x) \in \mathfrak{V}(x)$ such that

$$m \geq n, \quad V(x) \in \mathfrak{V}_m \text{ and } V(x) \subset U(x).$$

A metric space is a ranked space. Another examples of ranked spaces shall be found in the paper of K. Kunugi [1].

1.2. Convergence. Let $\{x_n\}$ be a sequence in a ranked space E. Now we shall consider a convergence introduced by K. Kunugi [2].

(1.2.1) **Definition.** We say that *a sequence* $\{x_n\}$ *converges to* x in a ranked space E, and we write $\{\lim_n x_n\} \ni x$ if and only if there exists a sequence $\{V_n(x)\}$ of neighborhoods and a sequence $\{\alpha_n\}$ of integers such that

$$V_0(x) \supset V_1(x) \supset V_2(x) \supset \cdots \supset V_n(x) \supset \cdots, \quad 0 \leq n < \omega_0,$$
$$\alpha_0 \leq \alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_n \leq \cdots, \quad 0 \leq n < \omega_0,$$
$$\sup_n \alpha_n = \omega_0, \quad V_n(x) \ni x_n, \text{ and } V_n(x) \in \mathfrak{V}_{\alpha_n}(x),$$

for $n=0, 1, 2, \cdots$.

If $\{\lim_n x_n\} \ni x$, we call x a *limit* of sequence $\{x_n\}$.

Then the following propositions hold:

(1.2.2) **Proposition.** *Let* $\{x_{n_i}\}$ *be an arbitrary subsequence of a sequence* $\{x_n\}$ *in a ranked space* E. *If* $\{\lim_n x_n\} \ni x$, *then*

$$\{\lim_i x_{n_i}\} \ni x.$$