78. *Axiom Systems of Aristotle Traditional Logic. IV*

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(Comm. by Kinjirô KUNUGI, M. J. A., May 13, 1968)

In this paper, we shall concern with the independence of axiom systems of Aristotle traditional logic. In my paper [2], new axiom systems of the Aristotle traditional logic are given as follows:

1 Aaa,

2 Iaa,

3 any one of AAA_1, AOO_2, OAO_3,

4 any one of AEE_4, EIO_4, IAI_4.

For the notations, see K. Iséki [1].

We shall prove that the above axiom systems are independent.

Let 1, 2, and 3 be values for terms. Let t and f be values for propositions, assuming the following:

$$Ctt=t,\quad Ctf=f,\quad Cft=t,\quad Cff=t,$$
$$Ktt=t,\quad Ktf=f,\quad Kft=f,\quad Kff=f,$$
$$Nt=f,\quad Nf=t,$$

where C, K, and N mean implication, conjunction and negation respectively.

First, we shall prove that *an axiom system* $<Aaa, Iaa, AAA_1,$ *any one of* $AEE_4, EIO_4, IAI_4>$ *is independent.*

Proof. Let $Aij=f$, $Iij=t$, then $Eij=f$, $Oij=NAij=t$, for every i, j $(i, j=1, 2, 3)$. Hence we have

$$Aaa=f,\quad Iaa=t,$$
$$AabAcaAcb=AijAkiAkj=fff=ff=t,$$
$$AabEbcEca=AijEjkEki=fff=ff=t,$$
$$EabIbcOca=EijIjkOki=ftt=ft=t,$$
$$IabAbcIca=IijAjkIki=tft=ft=t,$$

where $XabYbcZca$ means XYZ_4, and X, Y, Z denote categorical sentences. Hence Aaa is independent from other axioms.

Let $Iij=f$, $Aij=t$ $(i=j)$, f $(i\neq j)$, then $Eij=NIij=t$, $Oij=NAij$ $=f$ $(i=j)$, t $(i\neq j)$. Hence we have

$$Iaa=f,\quad Aaa=t$$
$$AabEbcEca=AijEjkEki=Aijtt=Aijt=t,$$
$$EabIbcOca=tfOca=fOca=t,$$
$$IabAbcIca=IijAjkIki=fAjkf=ff=t,$$

and $AabAcaAcb=AijAkiAkj$:

(i) $i=j$; $AiiAkiAki=tAkiAki=AkiAki=t$,

(ii) $i\neq j$; $AijAkiAkj=fAkiAkj=fAkj=t$.