176. On the Sets of Points in the Ranked Space. III

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In this paper, for a subset A of a ranked space R [1], we shall define two subsets of the ranked space, $\bar{A}$ and $\tilde{A}$. Both of them have some properties which are analogous to the closure in the usual topological space. We shall introduce several propositions with respect to $\bar{A}$ and $\tilde{A}$. We have used the same terminology as that introduced in the paper "On the sets of points in the ranked space II." [6].

Definition. Let A be a subset of a ranked space R. Then $\bar{A}$ and $\tilde{A}$ are defined as follows.

$$\bar{A}=\{x\,;\exists\{V_\alpha(x)\},\ V_\alpha(x)\cap A\neq\phi \text{ for all } \alpha\},$$
$$\tilde{A}=\{x\,;\forall\{V_\alpha(x)\},\ V_\alpha(x)\cap A\neq\phi \text{ for all } \alpha\},$$

where $\{V_\alpha(x)\}$ is a fundamental sequence of neighborhoods with respect to a point x of R [2] and α is a natural number. We say that $\bar{A}$ is an r-closure of A and that $\tilde{A}$ is a quasi r-closure of A.

Proposition 1. *If A is a subset of a ranked space R, then*

(1) $\tilde{A}\subseteq\bar{A}$,

(2) *if R satisfies Condition* (M) [3] *then* $\bar{A}=\tilde{A}$.

Proof. It is easy to prove (1).

If $p\in\bar{A}$, then by the definition there exists a fundamental sequence of neighborhoods of p, $\{V_\alpha(p)\}$, such that $V_\alpha(p)\cap A\neq\phi$ for all α.

Let $\{U_\beta(p)\}$ be an arbitrary fundamental sequence of neighborhoods of p, and $V_\alpha(p)\in\mathfrak{U}_{\gamma_\alpha}$ and $U_\beta(p)\in\mathfrak{U}_{\delta_\beta}$. Then for each β, there exists γ_α such that $\delta_\beta\leq\gamma_\alpha$. By Condition (M), $U_\beta(p)\supseteq V_\alpha(p)$, consequently $U_\beta(p)\cap A\neq\phi$. Therefore $p\in\tilde{A}$, that implies $\bar{A}\subseteq\tilde{A}$. Then, $\bar{A}=\tilde{A}$ because by (1) $\bar{A}\supseteq\tilde{A}$.

Remark 1. In general $\bar{A}\neq\tilde{A}$. For example, if $A=\{z_n\}$, where $\{z_n\}$ is a sequence of points in Example 1 [3], then $\bar{A}\neq\tilde{A}$.

Proposition 2. *If A and B are subsets of a ranked space, then*

(1) *if $A\subseteq B$, then $\bar{A}\subseteq\bar{B}$ and $\tilde{A}\subseteq\tilde{B}$,*

(2) *$A\subseteq\bar{A}$ and $A\subseteq\tilde{A}$,*

(3) *$\overline{A\cup B}=\bar{A}\cup\bar{B}$ and $\widetilde{A\cup B}=\tilde{A}\cup\tilde{B}$,*

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