208. *On Definitions of Commutative Rings*

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G. R. Blakley and S. Ôhashi, one of the present authors give some interesting axioms of commutative rings (see [1], [2]). In this note, we shall give axiom systems of commutative rings, and semirings.

We shall consider a system $\langle R, +, \cdot, -, 0, 1\rangle$, where R is a non-empty set, 0 and 1 are elements of R, $+$, and $\cdot$ are binary operations on R, and $-$ is a unary operation on R.

Theorem 1. $\langle R, +, \cdot, -, 0, 1\rangle$ *is a commutative ring, if it satisfies the following conditions*:

1) $r=0+r$,

2) $r\cdot 1=1\cdot r=r$,

3) $((-r)+r)\cdot a=0$,

4) $((c+(a\cdot y))+b)\cdot r=c\cdot r+(r\cdot b+a\cdot(y\cdot r))$.

As usual case, we omit the symbol $\cdot$ to write formulas. Therefore, ab means $a\cdot b$.

5) $(-r)+r=0$.

Proof.

$0=((-r)+r)1$ by 3),

$=(-r)+r$. by 2).

6) $0a=0$.

Proof.

$0a=((-r)+r)a$ by 5),

$=0$. by 3).

7) $a+b=b+a$.

Proof.

$a+b=((0+(a1))+b)1$ by 1), 2),

$=01+((1b)+a(11))$ by 4),

$=0+(b+a)$ by 2), 6),

$=b+a$. by 1).

8) $ab=ba$.

Proof.

$ab=(0+(00))+a)b$ by 1), 6),

$=0b+(ba+0(0b))$ by 4),

$=0+(ba+0)$ by 6),

$=0+(0+ba)$ by 7),

$=ba$. by 1).