17. *A Generalization of Prime Ideals in Rings. II**)

By Yoshiki KURATA and Sumiko KURATA
Dapartment of Mathematics, Yamaguchi University
and Ube Chūō High School

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1. Recently, generalizing the notions of prime ideals and primary ideals in rings, Murata, Kurata, and Marubayashi [1] have considered the notions of f-prime ideals and f-primary ideals in rings, and obtained, along with other things, the uniqueness theorem of f-primary decompositions of ideals, under certain assumptions.

Continued from [1], in this paper, we shall investigate the ideals which can be represented as the intersection of a finite number of f-primary ideals.

Let R be an arbitrary ring. Throughout this paper, ideals will always mean two-sided ideals in R and we shall assume the following conditions as same as in [1]:

(β) For any ideal A and any ideal B not contained in $r(A)$, we have $A:B\neq\varnothing$.

(γ) If S is an f-system with kernel S^*, and if, for any ideal A, $S\cap A$ is not empty, then so is $S^*\cap A$.

(δ) For any f-primary ideal Q, we have $Q:Q=R$.

2. Isolated components

Definition 1. Let A be an ideal and let S be an f-system. The isolated component A_S of A determined by S will be defined as follows:

$$A_S=\begin{cases}\bigcup_{s\in S}(A:s) & \text{(if } S \text{ is not empty)}\\ A & \text{(if } S \text{ is empty).}\end{cases}$$

For any f-system $S\neq\varnothing$, $C(S)$ is an f-prime ideal containing $r((0))$. If $s\in S$, then $s\notin r((0))$ and hence by the assumption (β) we have $(0):s\neq\varnothing$. This shows that $A:s$ and whence A_S is not empty. So, it can be proved similarly as in [1] that A_S is an ideal containing A.

Another characterization of f-primary ideals can be given by means of isolated components.

Proposition 2. *An ideal Q is f-primary if and only if, for any f-system S, either $Q_S=Q$* or *$Q_S=R$ holds.*

Proof. Suppose that Q is f-primary. If S is empty, then the assertion is trivial. Now we may suppose that there exists a non-

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