160. *On the Dimension of the Product of a Countably Paracompact Normal Space with the Unit Interval*

By Tatsuo GOTO

(Comm. by Kinjirô KUNUGI, M. J. A., Oct. 13, 1969)

1. Introduction. In 1953, K. Morita [3] proved that $\dim(X\times I)=\dim X+1$ holds if X is a paracompact Hausdorff space, where I denotes the closed unit interval [0, 1] and dim means the covering dimension. He also conjectured that the above equality would be valid if X is countably paracompact normal. In this note we shall answer this problem in the affirmative.

Let us denote by $D(X;G)$ the cohomological dimension of a space X with respect to an abelian group G, that is, $D(X;G)$ is the largest integer n such that $H^n(X, A;G)\neq 0$ for some closed set A of X, where H^* denotes the Čech cohomology based on all locally finite open coverings. We shall prove

Theorem 1. *Let X be a countably paracompact normal space with a finite covering dimension and G a countable abelian group. Then* $D(X\times I;G)=D(X;G)+1$.

As is proved by Y. Kodama [2], the above relation holds for any abelian group G if X is a paracompact Hausdorff space. If we take $G=$the group of integers Z in Theorem 1, we have $\dim(X\times I)=\dim X+1$, since $D(X;Z)=\dim X$ for each normal space X with a finite covering dimension.

2. Lemmas. The following lemmas are proved in [1].

Lemma 1. *Let X be a countably paracompact normal space and Y a compact metric space. Then the Künneth formula $H^n(X\times Y;G)\cong\sum_{p+q=n}H^p(X;H^q(Y;G))$ holds for each countable abelian group G.*

Lemma 2. *Let X, Y be countably paracompact normal spaces and let A, B be closed sets in X, Y respectively. If $f:(X, A)\to(Y, B)$ is a map such that*

(1) *$f|X-A: X-A\to Y-B$ is a onto homeomorphism;*

(2) *if F is a closed set in X and $F\subset X-A$, then $f(F)$ is closed in Y. Then $f^*: H^*(Y, B;G)\to H^*(X, A;G)$ is a onto isomorphism for each abelian group G.*

Let X be a normal space and A a closed set in X. By [2, Lemma 3] for each countable locally finite open covering $\mathfrak{U}$ of A, there exists a countable locally finite open covering $\mathfrak{V}$ of X such that $\mathfrak{V}|A$ is a refinement of $\mathfrak{U}$. Therefore if we denote by $H_c^*(X, A;G)$ the Čech