151. On Wiener Compactification of a Riemann Surface Associated with the Equation $\Delta u=pu$

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1. We consider an elliptic partial differential equation

$$(*) \qquad \Delta u=pu$$

on a Riemann surface R, where $\Delta=\partial^2/\partial x^2+\partial^2/\partial y^2$ and p is a non-negative and continuously differentiable function of local parameters z such that the expression $p(z)|dz|^2$ is invariant under the change of local parameters z. We call such a function p a density on R.

The investigation of the global theory of $(*)$ was begun by M. Ozawa [8] and continued by many others (for example, L. Myrberg [4], H. L. Royden [9], M. Nakai [5] [6] and F. Maeda [3]).

Associated with the equation $(*)$, Wiener functions and the Wiener compactification $R^*_{W^p}$ of R are discussed; more generally the Wiener compactification of harmonic spaces is studied by C. Constantinescu and A. Cornea [2]. In this note we shall examine how the Wiener compactification depends on a density p, and we shall give the following result (Theorem 4); If p and q are two densities on R satisfying

$$\text{(I)} \qquad \alpha^{-1}q\leq p\leq \alpha q$$

on R for some constant $\alpha\geq 1$, or

$$\text{(II)} \qquad \iint_R |p(z)-q(z)|\ dxdy<\infty$$

then there exists a homeomorphism Φ^* of $R^*_{W^p}$ onto $R^*_{W^q}$ such that $\Phi^*(\Gamma_{W^p})=\Gamma_{W^q}$, where Γ_{W^p} (or Γ_{W^q}) is a harmonic boundary of $R^*_{W^q}$ (or $R^*_{W^q}$).

2. Let Ω be an open subset of a Riemann surface R. A function u on Ω is called p-harmonic on Ω if u is twice continuously differentiable and satisfies $(*)$. A p-superharmonic function is defined as usual (see [3]). We know that a twice continuously differentiable function s on Ω is p-superharmonic on Ω if and only if $\Delta s-ps\leq 0$ on Ω. Let a be an arbitrary point on R. L. Myrberg [4] proved that if $p\not\equiv 0$, there exists always the Green function of R with pole at a for the equation $(*)$. We denote it by $g_a^{p,R}$.

3. A real-valued function f on R is called a p-Wiener function when f is quasicontinuous and has a p-superharmonic majorant and