12. *Ergodic and Mixing Properties of Measure Preserving Transformations*

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Totoki [6] has shown that strongly mixing Gaussian flows are all order mixing. As is well-known, the all order mixing implies the weak mixing and the weak mixing implies the ergodicity. Conversely, one can ask for which class of transformations ergodicity implies all order mixing. Halmos [2] has proved that if a continuous automorphism of a compact Abelian group is ergodic, then the automorphism is strongly mixing (i.e. 1-order mixing), and Rohlin [4] has proved further that every ergodic continuous automorphism of a compact Abelian group is all order mixing.

In this paper we study some classes of the transformations of which ergodicity and strong mixing imply all order mixing respectively. Our transformations were first topologically studied by Keynes and Robertson in their paper [1].

Let $(\Omega, \mathscr{B}, m)$ be a probability measure space and I be the set of all integers or real numbers. Consider a group G of homeomorphisms of I and for each $g \in G$, define an automorphism T_g of $(\bigotimes_{i\in I} \Omega, \bigotimes_{i\in I} \mathscr{B}, \bigotimes_{i\in I} m)$ as follows:

$$T_g(\omega_i \mid i \in I) = (\omega_{g(i)} \mid i \in I) \qquad (\omega_i \mid i \in I) \in \bigotimes_{i\in I} \Omega.$$

We call each T_g a *G-index automorphism*.

Definitions. (i) T_g is *ergodic* if for every $E, F \in \bigotimes_{i\in I} \mathscr{B}$ with positive measure, there exists a positive integer n such that

$$\bigotimes_{i\in I} m(T_g^n E \cap F) > 0.$$

(ii) T_g is *weakly mixing* if the product automorphism $T_g \otimes T_g$ is ergodic.

(iii) T_g is *strongly mixing* if for every $E, F \in \bigotimes_{i\in I} \mathscr{B}$ with positive measure,

$$\lim_{n\to\infty} \bigotimes_{i\in I} m(T_g^n E \cap F) = \bigotimes_{i\in I} m(E) \bigotimes_{i\in I} m(F).$$

Lemma 1. *Let $g \neq e$. If T_g is ergodic, then there exists a positive integer n such that $g^n(\alpha) \cap \beta = \emptyset$ holds for every finite subsets α, β of I.*

Proof. Suppose there exist finite subsets α, β of I such that $g^n(\alpha) \cap \beta = \emptyset$ for all n. Choosing $A, B \in \mathscr{B}$ with positive measure so