8. *Some Theorems on Cluster Sets of Set-Mappings*

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1. This is a résumé of the paper which will appear elsewhere [4]. A set-mapping F from a non-empty set A into a set B is, by definition, a mapping from A into the totality of subsets of B, so that, for every $a \in A$, $F(a)$ denotes a (possibly empty) subset of B. A non-trivial example known to complex variable analysts is, of course, a multiple-valued analytic function obtained by analytic continuation throughout a plane domain D starting with a fixed function element with the centre in D. This defines a set-mapping from D into the Riemann sphere. Now, let T and S be topological spaces and F be a set-mapping from a subset $U \neq \emptyset$ of T into S. Let $G \neq \emptyset$ be a subset of U and $t_0 \in \bar{G}$, here and elsewhere, "bar" means the closure in the considered spaces. Then the cluster set $C_G(F, t_0)$ of F at t_0 relative to G is defined by the following:

$$C_G(F, t_0) = \bigcap \overline{F(N \cap G)},$$

where the intersection is taken over all neighbourhoods N of t_0 in T with

$$F(N \cap G) = \bigcup_{t \in N \cap G} F(t).$$

If, in particular, T is the disk $|z| \leqq 1$, U is $|z| < 1$ and $e^{i\theta}$ is a point of $|z| = 1$, then the full cluster set $C_U(F, e^{i\theta})$, a curvilinear cluster set $C_\gamma(F, e^{i\theta})$, the radial cluster set $C_\rho(F, e^{i\theta})$ and an angular cluster set $C_\Delta(F, e^{i\theta})$ at $e^{i\theta}$ are the cluster sets corresponding respectively to $G = U$, a simple arc in U with the initial point in U and the terminal point $e^{i\theta}$, the radius drawn to $e^{i\theta}$ and an angular domain Δ in U with the vertex at $e^{i\theta}$.

2. Size of cluster sets. We consider the case where T and S are metrizable and S is compact.

Theorem 1. *Let F be an arbitrary set-mapping from a subset $U \neq \emptyset$ of T into S such that $F(t) \neq \emptyset$ for any point $t \in U$. Let $\Sigma \neq \emptyset$ be closed in S and let K be the boundary (in T) of U. We set, for every $t \in K$,*

$$f(t) = \sup \text{ (inf resp.) } \{\text{dis}(\Sigma, \alpha) ; \alpha \in C_U(F, t)\}.$$

Then f is an upper (lower resp.) semi-continuous function on K. We have the same conclusion if we replace $\text{dis}(\Sigma, \alpha)$ in the definition of f by

$$\overline{\text{dis}}(\Sigma, \alpha) = \sup \{\text{dis}(s, \alpha) ; s \in \Sigma\}.$$