33. *Continuous Affine Transformations of Locally Compact Totally Disconnected Groups*

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1. Introduction. In this paper the followings shall be proved. Let G be a locally compact totally disconnected non-discrete group and let T be a continuous automorphism of G. If there are two elements a and w in G such that $\{T(a)^n(w) \mid n=0, \pm 1, \pm 2, \cdots\}$ is dense in G then G is compact, where $T(a)$ is the continuous affine transformation of G defined by $T(a)(x)=a \cdot Tx$ for x in G. Next let G be a locally compact totally disconnected (not necessarily non-discrete) group and let T be a continuous automorphism of G such that there is an element w in G such that $\{T^n(w) \mid n=0, \pm 1, \pm 2, \cdots\}$ is dense in G. Then G is compact, whence T. S. Wu's problem (see [1, p. 518] and also [6]) raised in 1967 concerning the study of topology of a locally compact group G which admits an ergodic continuous automorphism with respect to a Haar measure on G is solved affirmatively.

Recently M. Rajagopalan and B. Schreiber [4] have proved that if a locally compact group G has a continuous automorphism which is ergodic with respect to a Haar measure on G then G is compact. In their proof the property of Fourier-Stieltjes coefficients of idempotent measures on the torus $K=\{\exp(i\theta) \mid 0 \leqq \theta < 2\pi\}$ plays an important role. In studying their techniques of the proof I have been led to that the techniques can be applied to the arguments of continuous affine transformations.

2. Continuous affine transformations. Throughout this paper, T and $T(a)$ will be denoted a continuous automorphism of a locally compact group G and a continuous affine transformation of G induced by a in G and T, respectively.

Lemma 1. *Let H be a complex Hilbert space, let A be a bounded operator and U_1, U_2 unitary operators on H. Then for given ξ and η in H there is a complex regular measure μ on the 2-dimensional torus $K \times K$ whose Fourier-Stieltjes transform is given by*

$$\hat{\mu}(m,n)=\langle A U_1^m \xi, U_2^n \eta \rangle, \qquad -\infty < m, n < \infty.$$

Proof. Let ρ_1 and ρ_2 denote spectral measures on $[0, 2\pi)$ for U_1 and U_2, respectively. For ξ, η in H we have