65. *On Some Boundary Properties of Harmonic Dirichlet Functions*

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Introduction. The fundamental potential functions on open Riemann surfaces like Green function, harmonic measure etc. show very specific boundary behaviors provided the surfaces have smooth boundaries. But on general surfaces the situation become complicated, actually one has first to define the boundary values or normal derivatives at the ideal boundary.

We have shown [5] that canonical potentials, especially harmonic measures assume constant values quasi-everywhere on each component of Kuramochi boundary. For other compactifications such a property was investigated afterwards by Kusunoki-Mori [6], Ikegami [4] and Watanabe [9] (cf. also Nakai-Sario [8]). At the same time it was inquired whether this boundary behavior would characterize those functions, however the question is still open. The purpose of this paper is to give some comments to this problem from the viewpoint of normal derivatives.

1. In the following we shall use some terminologies and notations without repetitions (cf. Constantinescu-Cornea [2] and Ahlfors-Sario [1]). Let R be a hyperbolic Riemann surface and R^* a resolutive compactification of R. Let ω_a be the harmonic measure of the ideal boundary $\Delta=R^*-R$ with respect to a point $a\in R$. The carrier of ω_a coincides with the harmonic boundary Δ_0 of Δ. For fixed $a=a_0$ we denote ω_{a_0} by ω, and by $L^2(\Delta)$ the Hilbert space of real-valued functions on Δ square integrable with respect to $d\omega$.

A resolutive compactification R^* is called D-normal (Maeda [7]) if every $u\in HD(R)$ (space of harmonic Dirichlet functions on R) can be expressed as

$$u(a)=\int_\Delta f\, d\omega_a=H_f(a)$$

with a resolutive function f on Δ. The compactifications of Wiener, Royden, Martin and Kuramochi are all D-normal and of type S. In the sequel we shall assume R^* is D-normal, unless otherwise stated. The mapping

$$u\to f$$

is linear and one-to-one (cf. [6]). We call f the *boundary value* (*or*