87. *Subnormal Weighted Shifts and the Halmos-Bram Criterion*

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The question of when a weighted shift is subnormal was treated by J. G. Stampfli in a paper called "which weighted shifts are subnormal" [1]. In that paper, an explicit matricial construction is given for the minimal normal extension, assuming the shift to be subnormal. An examination of when this construction is possible leads to conditions for subnormality in terms of the weights. The construction is not easy and the conditions are not especially transparent. However, the conditions he obtains enable him to answer certain questions such as the following: how many initial weights (of a subnormal shift) can be prescribed arbitrarily?

The purpose of this note is to give a criterion for subnormality rather different from Stampfli's. The questions he answers seem much more accessible from our point of view and reduce to elementary problems concerning the moments of measures. Professor Halmos has informed us that the connection between subnormality and moment sequences (our corollary) was noted previously by C. E. Berger but apparently this observation is unpublished.

In addition, the criterion given below bears on a general question relating to the well-known general criterion for subnormality due to Halmos and Bram (see [2]). Let's agree to call an infinite matrix $(a_{ij})_0^\infty$ nonnegative if all the principal finite sub-matrices $(a_{ij})_0^n$ are non-negative definite. A sequence of vectors $\{f_i\}_0^\infty$ will be called sub-normal if the matrix $(\langle W^j f_i, W^i f_j \rangle)_0^\infty$ is non-negative (all this is relative to a fixed operator W). The Halmos-Bram theorem says W is subnormal if and only if each sequence is. The practical drawback of this criterion is evident. Consequently, it would seem of interest to find a non-trivial class of operators for which subnormality could be checked by examining only a small number of sequences. The weighted shifts are such a family.

We let H be a separable Hilbert space. The spectral radius of a bounded operator T is $r(T)$. We will always assume that the weights of a weighted shift are positive.

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