120. *Some Remarks on Boundedness of Linear Transformations from Banach Spaces into Orlicz Spaces of Lebesgue-Bochner Measurable Functions*

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Let X be an abstract set.

Let R be the set of real numbers. Let R^+ be the set of non-negative reals. Let Y, Z and W be arbitrary Banach spaces.

Denote by $|\quad|$ the norm of any element of Y, Z or W.

A collection V (non-empty) of subsets of X is said to be a pre-ring of subsets of X whenever $A_1, A_2 \in V$ implies $A_1 \cap A_2 \in V$ and $A_1 | A_2$ can be written as a disjoint union of some finite collection of members of V.

Let V be a pre-ring of subsets of X.

A function $v: V \to R^+$ will be called a volume whenever for every countable family of disjoint sets $A_t \in V (t \in T)$ such that $A = \bigcup_{t \in T} A_t \in V$ we have $v(A) = \sum_T v(A_t)$.

Let v be a volume defined on V. We call the triple (X, V, v) a volume space. Denote by V_v^+ the collection $\{A \in V : v(A) > 0\}$.

In [1], is developed the basic theory of the space of Lebesgue-Bochner summable functions generated by the volume space (X, V, v). We denote this space by $L_1(v, Y)$; also we denote by $S(V, Y)$ the space of all V-simple functions with values in Y, i.e., functions $f: X \to Y$ of the form $f(x) = \sum_{i=1}^{n} y_i X_{A_i}(x)$ where $y_1, \cdots, y_n \in Y$ and $A_1, \cdots, A_n$ are disjoint members of V, and by $S^+(V)$ the set of non-negative members of $S(V, R)$.

Let $f: X \to Y$. We call f v-locally summable, denoted by $f \in L_1^{\mathrm{loc}}(v, Y)$, whenever for each $A \in V_v^+$, $X_A \cdot f \in L_1$. We endow $L_1^{\mathrm{loc}}(v, Y)$ with the locally convex topology generated by the family of seminorms $\{\|\quad\|_A : A \in V^+\}$ where

$$\|f\|_A = \|X_A \cdot f\|_{1,v}$$

Let (p, q) be a pair of real-valued functions defined on the interval $\langle 0, \infty)$ which satisfy the following conditions: (i) p is continuous, $p: \langle 0, \infty) \to \langle 0, \infty)$, and p is differentiable with derivative p' on $(0, \infty)$; (ii) p is a diffeomorphism of $(0, \infty)$ with itself such that $q(s)$

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