153. Absolute Summability by Logarithmic Method of Fourier Series

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1. Introduction and Theorems.

1.1. Let $\sum a_n$ be an infinite series and (s_n) be the sequence of partial sums. If the function

$$L(x)=\frac{-1}{\log(1-x)}\sum_{n=1}^{\infty}\frac{s_n x^n}{n} \tag{1}$$

is of bounded variation on an interval $(c,1)$, then the series $\sum a_n$ is said to be absolutely summable by logarithmic method or $|L|$-summable (see [1] and [2]).

Let f be an even integrable function with period 2π and its Fourier series be $\sum a_n \cos nx$. R. Mohanty and J. N. Patnaik [2] have proved the following

Theorem 1. *If the function*

$$\frac{1}{t\log(2\pi/t)}\int_t^{\pi}\frac{f(u)du}{2\sin u/2}=\frac{g(t)}{t\log(2\pi/t)} \tag{2}$$

is integrable in the interval $(0,\pi)$, *then the Fourier series of* f *is* $|L|$-*summable at the origin.*

Our first object of this paper is to give an alternative proof of this theorem.

1.2. Let (p_n) be a sequence of non-negative numbers such that

$$p(x)=\sum_{n=1}^{\infty}p_n x^n<\infty \quad \text{for} \quad 0<x<1.$$

If the function

$$P(x)=\frac{1}{p(x)}\sum_{n=1}^{\infty}p_n s_n x^n \tag{3}$$

is of bounded variation on an interval $(c,1)$ $(0<c<1)$, then we say that the series $\sum a_n$ is absolutely Perron summable or $|P|$-summable. According as $p_1=1$ or $p_n=1/n$, then $|P|$-summability reduces to $|A|$-summability or $|L|$-summability, respectively.

Theorem 1 is generalized as follows:

Theorem 2. *Suppose that* (i) *the sequence* (np_n) *is of bounded variation and that* (ii) *there is an* a, $0<a<1$, *such that*

$$(1-x)^a p(x)\downarrow \quad as \quad x\uparrow 1. \tag{4}$$

If $g(t)/tp(1-t)$ *is integrable in the interval* $(0,\pi)$, *then the Fourier series of* f *is* $|P|$-*summable at the origin.*