151. *Summability of Fourier Series*

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1. Introduction and Theorems.

1.1. Let $\sum a_n$ be an infinite series and (s_n) be the sequence of its partial sums. If

$$L(x)=\frac{1}{-\log(1-x)}\sum_{n=1}^{\infty} s_n x^n/n \to s \quad \text{as} \quad x \uparrow 1,$$

then the series $\sum a_n$ is said to be (L) summable to s. We shall consider a more general summability. Let (p_n) be a sequence of nonnegative numbers and suppose that the series $p(x)=\sum_{n=1}^{\infty} p_n x^n$ converges for all x, $0<x<1$ and $p(x) \uparrow \infty$ as $x \uparrow 1$. If

$$P(x)=\frac{1}{p(x)}\sum_{n=1}^{\infty} p_n s_n x^n \to s \quad \text{as} \quad x \uparrow 1,$$

then the series $\sum a_n$ is said to be (P) summable to s.

About (L) summability of Fourier series, M. Nanda ([1], cf. [2] and [3]) proved the

Theorem I. *If*

$$(1) \qquad g(t)=\int_t^{\pi} \varphi(u)u^{-1}du = o(\log 1/t) \quad \textit{as} \quad t \downarrow 0$$

where $\varphi(u)=f(x_0+u)+f(x_0-u)-2s$, *then the Fourier series of* f *is* (L) *summable to* s *at the point* x_0.

We shall generalize this theorem to (P) summability in the following form.

Theorem 1. *Suppose that the sequence* (np_n) *is monotone (non-decreasing or non-increasing) and concave or convex and that*

$$p(x)/(1-x)^2p'(x) \to \infty \quad \textit{as} \quad x \uparrow 1.$$

If

$$(2) \qquad \int_{1-x}^{\pi} G(t)t^{-3}dt = o(p(x)/(1-x)^2p'(x)) \quad \textit{as} \quad x \uparrow 1$$

where $G(t)=\int_0^t |g(u)|du$, *then the Fourier series of* f *is* (P) *summable to* s *at the point* x_0.

The condition (2) is the consequence of

$$(3) \qquad \int_0^x (p(t)/(1-t)^3p'(t))dt \leqq Ap(x)/(1-x)^2p'(x) \quad \text{as} \quad x \uparrow 1$$

and