146. *Generalized Fermat's Last Theorem and Regular Primes*

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1. Introduction.

According to Fermat's Last Theorem (FLT) the equation

$$x^n+y^n=z^n, \qquad n>2 \tag{1}$$

has no integral solution in non-zero integers. Gandhi [3] generalizing FLT, conjectured that the equation

$$x^n+y^n=cz^n \tag{2}$$

has no solution if $c\leq n$. Here x, y, z are non-zero unequal integers, c and n are also integers. Gandhi [3] proved his conjectures for several even powers and quoted a mass of results from literature to support his conjecture. The purpose of the present paper is to prove

Theorem 1. *The equation*

$$x^l+y^l=cz^l \tag{3}$$

has no integral solutions, where c is any integer prime to the regular prime $l>3$, $(\phi(c), l)=1$ and

$$c^{l-1}\not\equiv 1 \pmod{l^2} \qquad 2^{l-1}\not\equiv c^{l-1} \pmod{l^2}.$$

Here and in what follows $\phi(c)$ denotes Euler's function.

Consider $n=l$ in (2), l being a regular prime. Let $(c, l)=1$ and $(\phi(c), l)=1$. Then $c<l$ satisfies the condition $(\phi(c), l)=1$ hence in view of Theorem 1 and Maillet's result [9] that the equation $x^l+y^l=lz^l$ is impossible, Gandhi's conjecture is verified for a regular prime l for all such values of c, which satisfy

$$2^{l-1}\not\equiv c^{l-1} \pmod{l^2},\ c^{l-1}\not\equiv 1 \pmod{l^2}$$

Note that the truth of the theorem does not depend on particular values of x, y and z.

To prove Theorem 1, we shall discuss it under three cases.

First Case xyz prime to l
Second Case $xy\equiv 0 \pmod{l}$
Third Case $z\equiv 0 \pmod{l}$.

We note that the following theorem due to Györy [4], contains our theorem for the first two cases, hence we need to prove our theorem for third case only.

Theorem (Györy). *Let p be an arbitrary odd prime >3. If $(\phi(c), p)=1$, $c^{p-1}\not\equiv 2^{p-1} \pmod{p^2}$ then $x^p+y^p=cz^p$, $p\nmid z$ has a solution only if $r^{p-1}\equiv 1 \pmod{p^2}$ for an arbitrary divisor r of c.*

For other results for the diophantine equation $x^n+y^n=cz^n$, refer-