218. *On a Characterization of a Potential Theoretic Measure*

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(Comm. by Kinjirô KUNUGI, M. J. A., Nov. 12, 1970)

Introduction. In the first place, G. Anger [1] pointed out that continuous potentials play an important role in the theory of potential. We are concerned with a kernel $\phi(x,y)$ with continuous potentials in a locally compact space. In the case, we can define a certain family of potential theoretic positive measures $G^+(\phi)$ of which the adjoint potentials are integrable by all measures generating continuous potentials. The aim of this paper is to characterize the family of measures $G^+(\phi)$, which answers at the same time for a question posed by G. Anger [2] in the case that ϕ are Newtonian kernel Φ_N and a kernel Φ_W associated with the fundamental solution of the heat equation. For the Newtonian kernel Φ_N, H. Cartan [4] gave the following well known result; In order that a positive measure μ is an element of $G^+(\Phi_N)$, it is necessary and sufficient that the potential of μ is not identically infinity. But the above result does not hold for the kernel Φ_W and then we must find another characterization.

1. Notations and definitions. Let Ω be a locally compact Hausdorff space and $\phi(x,y)$ a measurable function in $\Omega\times\Omega$. The kernel $\check{\phi}(x,y)$ defined by $\check{\phi}(x,y)=\phi(y,x)$ is called the adjoint kernel. Setting $\phi^+(x,y)=\sup(\phi(x,y),0)$ and $\phi^-(x,y)=-\inf(\phi(x,y),0)$, we can denote $\phi(x,y)=\phi^+(x,y)-\phi^-(x,y)$. The ϕ-potential of a positive Radon measure μ in Ω is defined by

$$\phi\mu(x)=\int^*\phi(x,y)d\mu(y),$$

provided that $\phi\mu^+(x)$ and $\phi\mu^-(x)$ are not infinity at the same time. The adjoint potential $\check{\phi}\mu(x)$ is defined by the analogous way. $\phi(x,y)$ is called S-kernel if there exists at least such a positive measure λ that the support $S\lambda$ is compact and the potentials $\phi\lambda^+(x)$ and $\phi\lambda^-(x)$ are continuous in Ω. In the case that $\phi(x,y)$ is S-kernel, we can consider the following classes of measures,

$$F^+(\phi)=\{\lambda\,;\,\lambda\geqq 0, S\lambda \textit{ compact}, \phi\lambda^+(x) \textit{ and } \phi\lambda^-(x) \textit{ continuous in } \Omega\}$$

$$G^+(\phi)=\left\{\mu\,;\,\mu\geqq 0, \int^*\check{\phi}\mu^+d\lambda \textit{ and } \int^*\check{\phi}^-\mu d\lambda<+\infty \textit{ for any } \lambda\in F^+(\phi)\right\}.$$

$\phi(x,y)$ is called T-kernel if $\phi(x,y)$ is non-negative S-kernel and for

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