

216. Neutron Transport Process on Bounded Homogeneous Domain

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1. The neutron transport process has been studied by Harris ([1]) and Mullikin ([5]) as an application of the theory of discrete-time branching processes. The main problems are the asymptotic behavior of the number of neutrons, the extinction probability and the rate of convergence of the extinction probability at time t to the extinction probability. In this paper we consider similar problems for a mono-energetic and isotropic neutron transport process on a bounded homogeneous domain. We will formulate the model as a continuous-time branching process and apply the general theory of such processes ([2]). Main results are the theorems 1~5 below. It will be seen that the expected number of new-born neutrons plays an essential role in the above problems. This is a typical property of branching processes, which is well known for Galton-Watson processes.

2. Let D be a bounded closed convex domain in the three-dimensional Euclidian space R^3 with a smooth boundary and Ω be the unit sphere in R^3 . We denote by G the product space $D \times \Omega$ and ∂G the set (x, ω) where x belongs to the boundary of D and ω is a direction exiting the domain; i.e., $(\omega, n_x) \geq 0$ where n_x is the direction of the outer-normal at x . We formulate our model of neutron transport process as a continuous-time branching process as follows; a particle at $x \in D$ starting with unit speed in the direction ω^{*} will, at a random time T which is exponentially distributed with mean σ^{-1} , be absorbed, scattered, or multiplied by fission. If it leaves the domain D before T , then it is absorbed. The direction of new particles is supposed to be isotropically distributed. Each of new particles, independently each other, performs a similar motion as the original one. We can construct such a branching process on a suitable probability space ([2]) and every probabilistic argument below is based on this process.

Let $F[\xi] = \sum_{n=0}^{\infty} p_n \xi^n$ where p_n is the probability that n neutrons are produced when fission occurs. (In particular p_0 is the probability of absorption and p_1 the probability of scattering.) We will assume $F'[1] < \infty$ and $p_0 + p_1 < 1$. The first assumption guarantees that the

*) This statement will be simplified below as "starting at (x, ω) ."