213. A Remark on the Concept of Channels. II

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In the previous note [5], the concept of generalized channels is introduced. In the present note, the effect of the action of a motion on the input will be discussed. Incidentally, the deformation of the spectra of operators through a generalized channel will be considered.

1. Following after the notation of Dixmier [4], the subconjugate space A_* of a von Neumann algebra A is the Banach space of all ultraweakly continuous linear functionals defined on A. A *generalized channel* K is a positive linear transformation defined on a von Neumann algebra B, say *output*, with the range in a von Neumann algebra A, say *input*, which preserves the identity; in other words, the subconjugate K_* of K is positive and norm preserving:

$$\|K_*\rho\| = \|\rho\|, \tag{1}$$

for $\rho \geqq 0$, cf. [5]. Conveniently, K_* will be called a generalized channel too. A generalized channel K_* transfers a normal state ρ from A_* to B_*, and $K_*\rho$ is a normal state of B. If $A = B$, then a generalized channel K will be called a *transition*; if A is abelian then a transition is a transition operator in probability.

2. The concept of generalized channels is born on the information theory, but it is not restricted. Suppose that the input A represents a physical system and the output an observation instrument. A state of the physical system will drive some state of the instrument, if they are connected together. Thus a generalized channel can be considered as a mathematical model for physical measurements. Especially, the situation is suitable for statistical mechanics, including both classical and quantum.

A *motion* μ of a system A is a ($*$-preserving) automorphism of a von Neumann algebra A, according to a modification of the definition of Segal [8]. A motion μ is ultraweakly continuous; hence the subconjugate (may be abbreviated by μ too) of the motion transforms a normal state ρ to a normal state ρ^μ by

$$\rho^\mu(a) = \rho(a^\mu), \tag{2}$$

for every $a \in A$.

What happens for the receiver if a motion acts on input? The observer obtained $K_*\rho$ before the motion through the channel K. After the motion, he receives $K_*\rho^\mu$. Put